



ACE

Engineering College

(NBA ACCREDITED B.TECH COURSES & NAAC "A" GRADE)

An Autonomous Institution

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DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

I B.Tech I Semester Regular(R20) (2020-21)

Sub:M-1

ASSIGNMENT 1:

1. Find the determinant of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$

2. Which of the following matrices are symmetric or skew –symmetric

i) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 4 & -5 \\ -4 & 2 & -6 \\ -5 & 6 & 3 \end{bmatrix}$

3. Express the matrix as the sum of a symmetric and skew-symmetric matrix.

$$\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

4. Find the ranks of the following matrices using elementary row transformation

i) $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

ii)
$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ 3 & 3 & 6 & 6 & 12 \end{bmatrix}$$

5. For what value of k , the matrix A has ranks

$$\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$$

6. Find the ranks of the following matrices by reducing them to the normal form

i)

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

7. Which of the following matrices are orthogonal? What is the value of the determinant if it is an orthogonal matrix?

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

8. Show that the following matrices are orthogonal

$$\begin{bmatrix} 2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$$

9. Find the number of linearly independent vectors out of the following

a) $V_1 = (1, 1, 3, 2)$

$$V_2 = (2, 3, 4, 5)$$

$$V_3 = (5, 7, 11, 12)$$

10. Find P and Q such that PAQ is in normal form Where A is given by

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

11. Find whether the following equations are consistent

i) $x + 2y - z = 3; 3x - y + z = -1; 2x - 2y + 3z = 2$

ii) $x_1 + x_2 + x_3 + x_4 = 0; x_1 + x_2 + x_3 + x_4 = 4$

$$x_1 + x_2 + x_3 + x_4 = -4; x_1 + x_2 + x_3 + x_4 = 2$$

12. Find P, Q so that the following equations have i) no solution ii) unique solution infinite no. of solutions.

$$2x + 3y + 5z = 9, 7x + 3y + 2z = 8, 2x + 3y + pz = q$$

13. Solve the system of homogeneous equations

$$\text{i) } x+y+w=0, y+z=0, x+y+z+w=0,$$

$$x+y+2z=0$$

14. For what values of k the system of equations will have a non trivial solution

$$2x+3ky+(3k+4)Z=0$$

$$x+(k+4)y+(4k+2)z=0$$

$$x+2(k+1)y+(3k+4)z=0$$

15. Determine the values of λ for which equations have non-trivial solutions and solve them in each case.

$$\text{i) } x+2y+3z= \lambda x;$$

$$3x+y+2z= \lambda y$$

$$2x+3y+z= \lambda z$$

ASSIGNMENT 2:

1. Find the characteristic roots of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

2. Find the eigen values and the corresponding eigen vectors of the following matrices

i) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

ii)

$$\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$

iii)

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

3. Prove that sum of eigen values of A is equal to trace of A and product of the eigen values equal to $\det A$.

4. Verify that the sum of the eigen values is equal to the trace of A for the

Matrix $A =$

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

5. Verify the Cayley-Hamilton Theorem and hence find A^{-1} in the following

terms $A =$

$$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

6. Determine the eigen values and eigen vectors of $B = 2A^2 - \frac{1}{2}A + 3I$ where A

$$\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$$

7. Let λ be an eigen value of a square matrix A and x be its corresponding eigen vector. Then verify the following properties of A .

a. $(1/\lambda)$ is an eigen value of A^{-1} .

b. The determinant of A is the product of all its eigen values.

8. Diagonalize the following matrices i) $A =$

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

and hence find A^5

$$\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

9. Show that the matrix A=

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

10a) Prove that two eigen vectors of a real symmetric matrix are orthogonal.

b) Prove that the eigen values of a real symmetric matrix are real.

c) Prove that the eigen values of a real skew symmetric matrix are either zero or purely imaginary.

d) Prove that the two eigen vectors corresponding to the two different eigen values are linearly independent.

e) Show that the eigen values of a unitary matrix are of unit modules.

f) Prove that the inverse of a unitary matrix is unitary.

11. By the use of an orthogonal transformation, reduce the quadratic forms into their canonical forms and find their nature, index and signature.

a) $3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$

b) $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$

c) $X_1^2 + 2X_2^2 + 2X_3^2 - 2X_1X_2 + 2X_2X_3 - 2X_1X_3$

d) $5x^2 + 26y^2 + 10z^2 + 4yz + 14zx - 6xy$

$$e) 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$$