R13

Code No: 113BT

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, December-2014

PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

Part- A **(25 Marks)**

1.a) What are the axioms of probability? [2M]

b) Explain the types of Random variables. [3M]

c) Write the properties of probability distribution function? [2M]

d) A Continuous random variable X has a probability density function given by **$f(x) = 3x^2$ $0 \le x \le 1$.** Find **a** such that $P(X \le a) = P(X > a)$. [3M]

e) Define Marginal distribution & Marginal density Functions. [2M]

f) Statistically independent Random Variables X and Y have moments $m_{10}=2$, $m_{20}=14$, $m_{02}=12$, $m_{11}=-6$. Find μ_{22}. [3M]

g) State any 2 properties of Cross Correlation Function. [2M]

h) For the given auto correlation function for a stationary process is $Rxx(\tau) = 25 + \frac{4}{1+6\tau^2}$ Find the mean and variance. [3M]

i) State any 2 properties of the power density spectrum? [2M]

j) Write Wiener-Khinchine relations? [3M]

Part-B **(50 Marks)**

2.a) A binary communication channel carries data as one of the two types of signals denoted by 0 and 1. Owing to noise a transmitted 0 is sometimes received as 1 and a transmitted 1 is sometimes received as a 0. For a given channel, assume a probability of 0.94 that a transmitted 0 is correctly received as a 0 and a probability of 0.91 that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transmitting a 0. If a signal is sent, Determine

(i) Probability that a 1 is received.

(ii) Probability that a 0 was received

(iii) Probability that a 1 was transmitted, given that a 1 was received

(iv) Probability that a 0 was transmitted, given that a 0 was received

b) State Random Variable with suitable example.

OR

3.a) State and Prove the Bayes theorem of probability.

b) Let A_1, A_2, A_3 are 3 mutually exclusive and exhaustive events associated with experiment E_1.B_1, B_2, B_3 are 3 mutually exclusive and exhaustive events associated with experiment E_2.

	B_1	B_2	B_3	p(Aj)
A_1	3/36	*	5/36	*
A_2	5/36	4/36	5/36	14/36
A_3	*	6/36	*	*
P(Bj)	12/36	14/36	*	*

i. Find missing Probabilities
ii. $P(B_3/A_1)$ and $P(A_1/B_3)$
iii. A_1,B_1 are independent or not

4.a) Explain any 4 Properties of Probability Distribution Function.
b) A Continuous random variable X defined by a probability density function given by **$f(x) = 5(1-x^4)/4 \quad 0 \le x \le 1$.** Find E[X], $E[X^2]$ and Variance.

OR

5.a) Find mean and variance of Uniform Random Variable.
b) Find the characteristic function and first moment for
$f_X(x) = (1/b)\exp(-(x-a)/b) \quad x \ge a$
$= 0 \quad$ else

6.a) Following table represent the joint probability density function

↓Y.

X→

	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

(i). Evaluate the marginal distribution of X and Y.
(ii). Conditional distribution of X given Y=2.
(iii). Conditional distribution of Y given X=3.
(iv). $P(X \le 2, Y=3)$, $P(Y \le 2)$, $P(X+Y<4)$

b) Two random variables X and Y have joint characteristic function
$\Phi_{X,Y}(\omega_1,\omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$
Show that X and Y are zero mean random variables and uncorrelated

OR

7.a) Two random variables Y_1 and Y_2 related to arbitrary random variables X and Y by co-ordinate rotation $Y_1 = X\cos\theta + Y\sin\theta$, $Y_2 = -X\sin\theta + Y\cos\theta$.
i) Find the covariance function of Y_1 and Y_2.
ii) For what value of θ, the random variables Y_1 and Y_2 are uncorrelated
b) The joint probability density function of f(x,y) is given by
$f(x,y) = Ae^{-(x+y)} \quad 0 \le x \le y,\ 0 \le y \le \infty$
(i). Find the value of A
(ii). Find the marginal density of X and Y.
(iii). Verify that whether X and Y are independent.

8.a) Statistically independent zero mean Random process X(t) and Y(t) having auto correlation function $R_{XX}(\tau)=\exp(-|\tau|)$ and $R_{YY}(\tau)=\text{Cos}(2\pi\tau)$ respectively. Find Cross correlation function(CCF) of $W_1(t)$ and $W_2(t)$ if

$$W_1(t) = X(t) + Y(t) \text{ and } W_2(t) = X(t) - Y(t)$$

b) State any 4 Properties of Auto Correlation Function

OR

9.a) Prove that random process $X(t)=A\,\text{Cos}(\omega_c t+\theta)$ is a wide sense stationary process if it is assumed that A, ω_c are constants and θ is uniformly distributed over interval $0\leq\theta\leq2\pi$

b) Derive the Mean & Mean –Squared value of output response of a linear system.

10.a) Explain any 4 Properties of Power Density Spectrum.

b) Derive the power density spectrum of output of a system, in terms of its input PSD.

OR

11.a) Derive the relationship between Cross PSD & Cross Correlation Function.

b) The PSD of random process is given

$$S_{XX}(\omega) = \begin{cases} \pi, & |\omega| < 1 \\ 0, & elsewhere \end{cases}$$

Find its Autocorrelation function.

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