18/8/25

## UNIT - I

## Stresses & Strain

Engg.

1) ~~Fluid~~ Mechanics :- It deals with Study of forces and their effects on rigid body (fixied body).

Ex:- Displacement, velocity, acceleration etc...

2) Solid Mechanics (or) Strength of Materials :- It deals with Study of force and their effects on deformable bodies.

Ex:- Stress, Strain etc.

3) fluid Mechanics :- It deals with Study of forces and their effects on fluid.

Stress :- The internal resistance offered by a body against deformation when an external load is applied. It is denoted by "$\sigma$".

• Mathematical it is caliculated as load by Area.

$$\sigma = \frac{P}{A} = \frac{\text{Load}}{\text{Area}}$$

units :- Stress will be zero when body moves from one point to another point.

$$\sigma = \frac{P}{A} = \frac{N}{mm^2} \text{ (or) } \frac{KN}{m^2}$$

$$= \frac{10^3 N}{((1000) mm)^2}$$

$$= \frac{10^3 N}{(10^3)^2 mm^2}$$

$$\boxed{M \rightarrow 10^6 \quad G \rightarrow 10^9}$$

$$\boxed{1 KN/m^2 = 10^{-3} N/mm^2}$$

$$1 Mpa = 1 N/mm^2 = \frac{1N}{10^{-6} m^2}$$

$$\boxed{1 Mpa = 10^6 N/m^2 \text{ (or) } N/mm^2}$$

$$1\,GPa = 10^9 \frac{N}{m^2} = \frac{10^9 \times N}{10^6\,mm^2}$$

$$\boxed{1\,Gpa = 10^3\,N/mm^2}$$

| | |
|---|---|
| 1) $1\,GPa = 10^3\,N/mm^2$ | 2) $1\,MPa = 1\,N/mm^2$ |
| 3) $1\,GPa = 10^9\,N/m^2$ | 4) $1\,MPa = 10^6\,N/m^2$ |

Types of Stress :-

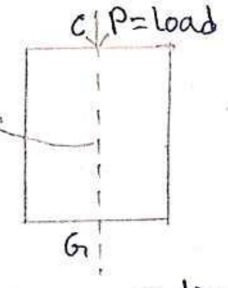
Stress
- Normal Stress
  - Direct Stress $= \frac{P}{A}$
  - Bending stress $= \frac{P\times e}{Z}$ (or) $\frac{M}{Z}$
- Shear stress / Tangential stress
  - Direct Shear stress
  - Torsional Shear Stress

1) Normal Stress :- Normal stress are always normal (perpendicular to the cross section). These are two types.

a) Direct Stress :- These stress are produced when the stresses is acted at C.G of cross section for prismatic body with axial loading. $\boxed{\sigma = \frac{P}{A}}$

axial loading

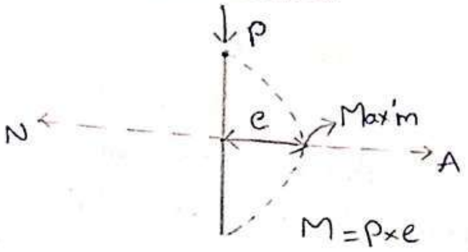
Load carries with C.G of c/s.




-> Direct stress are uniform across the cross section. Therefore, it can be calculated as Load / Area = P/A.

b) Bending Stress :- Bending stresses are produced by Bending movement these vary linearly from zero at one end to max at another end. (nutral axis).

$$\boxed{\sigma = \frac{M}{Z}}$$




2) Shear Stress / Tangential Stress :- Shear Stresses are resistances are offered by material against Shearing force. It's Value is determine by dividing the Shear force in the plane of Section by corresponding area.

a) Direct Shear Stress :- They are produced due to Shear forces acting on the Section.

b) Torsional Shear stresses :- These Stresses are produced when member is Subject to torsional movement (or) torque.

Types of Strain :-

1) Normal Strain :- It is due to normal Stress. It is denoted by 'e' (or) 'ϵ'.

- In other words it is defined as the ratio of change in dimension to the original dimension.

2) axial Strain :- It is defined as Strain produced in the direct of applied force is called as axial Strain. It is the ratio of change in linear dimension to the original dimension.

$$e_L \text{ (or) } \epsilon_L = \frac{\text{change in linear dimension}}{\text{Original Linear dimension}}$$

3, Lateral Strain :- Strain produced in the direction perpendicular to the direction of applied force. It is ratio of change in lateral dimension to the original dimension.

$$e_{later} = \frac{\text{Change in lateral dimension}}{\text{Original lateral dimension}} = \frac{\delta d}{d}$$

4, Shear Strain :- ($\phi$) Shear Strain are angular dimensions caused by Shearing force. It is denoted by "$\phi$".

$$\text{Shearing Strain} = \frac{\Delta\phi}{\phi}$$

5, Volumetric Strain :- It is defined as the ratio of change in volume to the original volume. It is dentod by ($\epsilon_v$) (or) $e_v$.

$$\epsilon_v = e_v = \frac{\text{Change in volume}}{\text{original volume}}$$

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Elasticity :- It is the property of Solid materials which regains it's original shape, ~~Corse~~ crosectic & Size after the removal of applied force.

Ex:- Rubber.

Plasticity :- It is the property of Solid materials which doesn't regains it's original Shape, crosection, & Size after the removal of applied force.

Ex:- metals.

Brittleness :- It is the tendency of a material to fracture or break with little or no plastic deformation.

Ex:- Glass, wood, Concrete, Caste iron etc.

**Ductility:-** Ductility is the ability of a material to undergo plastic deformation (permanent deformation) under tensile stress before fracture.

- Ductile materials can be drawn into wires or stretched without breaking.

Ex:- Copper, Gold, Aluminium, Steel etc.

**Ultimate stress:-** The maximum stress a material can withstand before fracture or rupture.

**Yield stress:-** The stress at which a material begins to deform permanently.

**Hooke's law:-** It states under direct loading with the proportionality limit, stress is directly proportional to strain.

$$\sigma \propto \epsilon$$

$$\sigma = E\epsilon$$

$$\boxed{E = \frac{\sigma}{\epsilon}}$$

$\sigma \rightarrow N/mm^2$

$\epsilon \rightarrow$ No units

$E \rightarrow N/mm^2$ $\boxed{E = 2\times10^5 N/mm^2}$

Where, E is a constant of proportionality which is called as Young's Modulus of Elasticity.

- Hooke's law is valid upto limit of proportionality.
- For Mild Steel proportionality limit & elastic limit are almost equal. But for other materials, elastic limit is higher than proportional limit.
- The slope of stress strain curve is known as modulus of elasticity.
- The standard value of steel is $E = 2\times10^5 N/mm^2$.

## Assumptions in Hooke's law :-

- Material is homogenous (properties are equal at all Points).
- Materal is Isotropic (properties are equal in all directions).
- Material is Elastic.

## Factor of Safety & Working Stress :-

It is defined as ratio of maximum stress to the Permissible Stress or working Stress. $\frac{act}{S.r} > 1 \Rightarrow \boxed{\frac{act}{str} > \frac{str}{req}}$

Fos = ~~Autual~~ Actual Strength / strength Required. (Fos > 1).

## For Brittle Materials :-

Fos = (ultimate stress or failure stress) / (working Stress or permissible Stress)

## For Ductile Materials :-

Fos = Yield stress / (working Stress (or) permissible Stress)

The maximum Stress or load Which a material can resist is called working Stress. The stress is enhanced by 2 or 3 times and that load is known as ultimate load.

## Significance of factor of Safety :-

- For the Safety of Structure before failure occurs, we use Fos so that under any Circumstances the structure will not fail.
- Fos should be always greater than 1, if it is not greater than 1, the structure may fail Suddenly without giving any notice to you.
- In Aircraft design generally margin of Safety is Considered rather than factor of Safety, and is Calculated as Fos-1.

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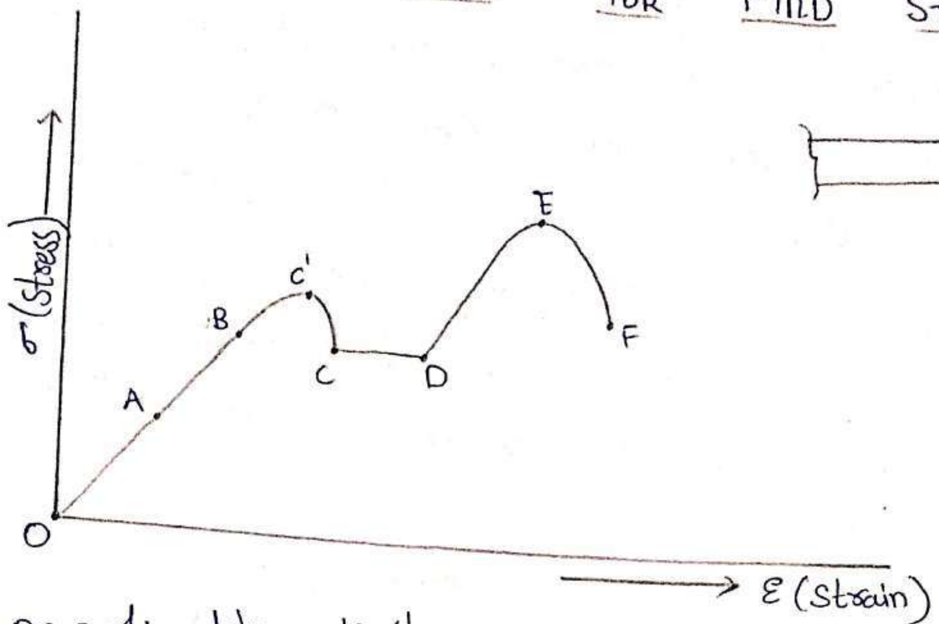
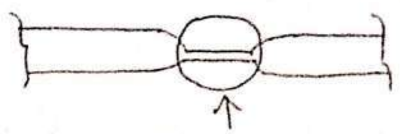
## *** STRESS - STRAIN CURVE FOR MILD STEEL :-




Cup & cone failure for Ductile materials

A = proportionality limit
B = Elastic limit
C' = upper yeild point
C = lower yeild point
CD = Strain hardening Zone
CDE = yeild plateau
E = ultimate Stress
F = Breaking / Fracture point
EF = Necking region

- UTM (universal ~~Tech~~ Testing Machine) can be used for tension test, Compression test, Bending test and Shear test.
- The Standard testing was done on a Standard Specimen having gauge length (effective length) to 2 feet and gauge die should be 0.5 feet by ASTM (Amercian Socity of testing materials).

**Point A :-**
It is called as proportional limit. The curve OA is linear and obeys Hook's law. Which is valid upto this limit only.

**Point B :-**
It is called elastic limit. After unloading within the point B, the strain can be released and this Specimen will regain it's original position.

Point $c'$ & $c$ :- $c'$ is equal to upper yield point & $c$ is equal to lower yield point. The fall of Stress from $c'$ to $c$ is due to the Slipping of carbon atoms in the molecular Structure of $c$.

Note :- Since, the mild Steel is Example of low Carbon Steel therefore, many no. of voids are un-occuipied hence, Carbon easily from one void to another. therefore, there are two yield points for mild Steel.

Region CD :- This is called yield plateau Zone region. From point C mild Steel behaves has plastic Since, without change in Stress, Strain is continuous therefore, the Stress corresponding to C is called as actual yield Stress.

Region DE :- This is called Strain harding Zone. In this region due to change in molecular Structure Since, distortion comes in the molecular Structure due to mixing of atoms therefore, the movement of the atom get's resistance. Hence, the Strength of the Specimen.

Point E :- This is called ultimate point, and the corresponding Stress is ultimate stress it represents the max strength of the Specimen on Stress - Strain diagram.

Region EF :- This is called as Necking region. This region Since, the Cross Section of the Specimen decreases rapidly therefore, Strength also decreases rapidly.

Point F :- F is called Fracture or breaking point.

Note :- Ductile materials are those Which has very large plastic

Strain after it's elastic limit. Brittle materials are those which has very low plastic Strain after it's elastic limit.

Elastic Constent (E) :-

i) Modulus of Elasticity (E) (or) Bending Modulus :- It is defined as (the ratio of normal stress to the normal strain. It denoted by 'E'.

$$\boxed{E = \frac{\sigma}{\epsilon}} = \frac{\text{Stress}}{\text{Strain}} = N/mm^2$$

ii) Modulus of Rigidity (or) Shear Modulus (G) (N,C) :-

It is defined as the ratio of Shear Stress to the Shear Strain.

$$\boxed{G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\phi}} = N/mm^2$$

iii) Bulk Modulus (K) :- It is defined as the ratio of compressive stress to the volumetric strain.

$$\boxed{K = \frac{\text{Compressive stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\epsilon_v}} = N/mm^2$$

iv) Poission's Ratio ($\mu$) :- It is defined as the ratio of lateral strain to the longitudinal strain.

$$\mu = \left| \frac{\text{Laterial Strain}}{\text{Longitudinal Strain}} \right|$$

- For all ductile materials 0 to 0.5 is the poission's ratio.

26/8/25 RELATIONSHIP BETWEEN ELASTIC CONSTANT :- (E, G, K, $\mu$)

i) $E = 2G(1+\mu) \rightarrow (1)$

ii) $E = 3K(1-2\mu) \rightarrow (2)$

iii) Eliminate '$\mu$' from (1) & (2)

$2G(1+\mu) = 3K(1-2\mu)$

$2G + 2G\mu = 3K - 6K\mu$

$$2G + 2G\mu = 3K - 6K\mu$$

$$6K\mu + 2G\mu = 3K - 2G$$

$$\mu\,(6K + 2G) = 3K - 2G$$

$$\boxed{\mu = \frac{3K - 2G}{(6K + 2G)}} \longrightarrow ③$$

from eq ①

$$E = 2G\,(1 + \mu)$$

$$E = 2G\left(1 + \frac{3K - 2G}{6K + 2G}\right)$$

$$E = 2G\left(\frac{6K + \cancel{2G} + 3K - \cancel{2G}}{6K + 2G}\right)$$

$$E = 2G\left(\frac{6K + 3K}{6K + 2G}\right)$$

$$E = \cancel{2}G\left(\frac{9K}{\cancel{2}(3K + G)}\right)$$

$$\boxed{E = \frac{9KG}{3K + G}}$$

Q) A rod 150cm long and of diameter 2cm is Subjected to an axial pull of 20kN. If Modulus of elasticity of the material of the rod $2 \times 10^5\ N/mm^2$. Determine i) The Stress ii) Strain iii) elongation of the rod.

Sol given data, Long = 150cm = 1500mm
Diameter = 2cm = 20mm
$P = 20kN = 20 \times 10^3 N$
$E = 2 \times 10^5\ N/mm^2$

i) Stress $= \frac{P}{A}$ ; $A = \frac{\pi}{4}(20)^2 = 314.15\ mm^2$

$\sigma = \frac{20 \times 10^3}{314.15}$ ; $\sigma = 63.66\ N/mm^2$

ii) Strain ($\epsilon$)

$$E = \frac{\sigma}{\epsilon}$$

$$2\times10^5 = \frac{63.66}{\epsilon}$$

$$\epsilon = \frac{63.66}{2\times10^5}$$

$$\epsilon = 3.183 \times 10^{-4} \cancel{N/m}$$

iii) Elongation of the rod :-

$$\epsilon = \frac{\Delta l}{l}$$

$$3.183\times10^{-4} = \frac{\Delta l}{1500}$$

$$\Delta l = 3.183\times10^{-4}\times1500$$

$$\Delta l = 0.477 \text{ mm}$$

Q) Find the Young's Modulus of a brass rod of diameter 25mm and of length 250mm which is subjected to a tensile load of 50kN. When the extension of the rod is equal to 0.3mm.

Sol G.D :- $L = 250\text{mm}$ ; $\phi \cdot d = 25\text{mm}$

$P = 50\text{kN}$ ; $\Delta l = 0.3\text{mm}$

i) Stress :- $\sigma = \frac{P}{A} = \frac{50\times10^3 \text{ N}}{\frac{\pi}{4}(25)^2 \text{ mm}^2}$

$$= 101.85 \text{ N/mm}^2$$

ii) Strain :- $\epsilon = \frac{\Delta l}{l} = \frac{0.3 \text{ mm}}{250 \text{ mm}}$

$$= 1.2\times10^{-3}$$

iii) (E) Young's Modulus :- $E = \frac{\sigma}{\epsilon}$

$$E = \frac{101.85 \text{ N/mm}^2}{1.2\times10^{-3}}$$

$$E = 84.87\times10^3 \text{ N/mm}^2$$

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Q) A ~~Steel~~ Determine the change in length, breadth and thickness of a Steel bar which is 4m long, 30mm wide and 20mm thick is subjected to an axial pull of 30kN in the direction of it length. take $E = 2\times10^5 N/mm^2$ and poission ratio $(\mu) = 0.3$

Sol G.D:- $L = 4m = 4\times10^3 mm$ ; $\mu = 0.3$

$w = 30mm$ ; $P = 30\times10^3 N$

$t = 20mm$

$E = 2\times10^5 N/mm^2$

Stress :- $\frac{P}{A}$

$\sigma = \frac{30\times10^3}{\cancel{4\times10^3\times30\times20}\ 30\times20}$

$\sigma = \cancel{0.25 N/mm^2}\ 50 N/mm^2$

Strain :- $(\epsilon)$

$\epsilon = \frac{\sigma}{E}$

$\epsilon = \frac{\cancel{0.25}\ 50}{2\times10^5}$

$\epsilon = \cancel{1.25\times10^{-6}}\ 2.5\times10^{-4}$

i) change in length $\delta L = \cancel{1.25\times10^{-6}\times4000} = 2.5\times10^{-4}\times4000$

$\delta L = \cancel{6\times10^{-3}mm}\ 5\times10^{-3}mm = 1mm$

$$\mu = \frac{\text{Lateral Strain}}{\text{longitudinal strain}}$$

$$\text{Lateral Strain} = 0.3 \times 2.5 \times 10^{-4}$$
$$= 7.5 \times 10^{-5}$$

$$\text{Lat. Strain} = \frac{\delta b}{b}$$

$$\delta b = \text{Lat. Strain} \times b$$
$$\delta b = 7.5 \times 10^{-5} \times 30$$
$$\delta b = 2.25 \times 10^{-3} \text{ mm}$$

$$\text{Lat. strain} = \frac{\delta t}{t}$$

$$\delta t = 7.5 \times 10^{-5} \times 20$$
$$\delta t = 1.5 \times 10^{-3} \text{ mm}$$

01/09/25

Q) A modulus of rigidity of a material is $0.5 \times 10^5 \, N/mm^2$. A 12mm diameter of rod of the material was subjected to an axial force of 14kN and the change in diameter was observed to be $3.6 \times 10^{-3}$ mm. Calculate the poision, Modulus of elasticity and bulk modulus of material.

Sol G.D :-

$G = 0.5 \times 10^5$ ; $d = 12mm$

$P = 14 \times 10^3 N$ ; $\delta d = 3.6 \times 10^{-3} mm$

$\mu = ?$ ; $E = ?$ ; $k = ?$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Lateral strain} = \frac{\delta d}{d}$$

$$= \frac{3.6\times10^{-3}}{12}$$

$$= 3\times10^{-4}.$$

$$\mu = \frac{3\times10^{-4}}{e_{long}} \rightarrow (1)$$

$$E = \frac{\sigma}{e_{Long}}$$

$$\text{Stress} = \frac{14\times10^{3}}{\frac{\pi}{4}(12)^2}$$

$$\sigma = 123.78\ N/mm^2$$

$$E = \frac{123.78}{e_{long}} \Rightarrow e_{long} = \frac{123.78}{E} \rightarrow (2)$$

$$E = 2G(1+\mu) \rightarrow (3)$$

$$E = 2\times0.5\times10^5\left(1+\frac{3\times10^{-4}\times E}{123.78}\right)$$

$$E = 1\times10^5\left(1+2.42\times10^{-6}E\right)$$

$$\boxed{E = 1.3\times10^5\ N/mm^2}$$

$$E = 2G(1+\mu)$$

$$1.3\times10^5 = 2\times0.5\times10^5(1+\mu)$$

$$\boxed{\mu = 0.3}$$

$$E = 3K(1-2\mu)$$

$$1.3\times10^5 = 3\times K(1-2\times0.3)$$

$1.3 \times 10^5 = 3K(1 - 2 \times 0.3)$

$\boxed{K = 1.08 \times 10^5 \, N/mm^2}$

Q) A Circular pipe of internal diameter of 45mm and thickness 5.5mm is Subjected to a force of 50kN and elongation was measured as 1.75mm. If the length of the pipe is 2.75m. Find the value of Young's Modulus of elasticity and the Stress.

So) G.D :- $d = 45mm$ ; $t = 5.5mm$ ; $\delta L = 1.75mm$

$\delta d = 1.75mm$ ; $L = 2.75m = 2750mm$ ; $P = 50 \times 10^3 N$

$E = ?$ ; $\sigma = ?$

$$D = 2t + d$$
$$= 2(5.5) + 45$$
$$= 56mm$$

$$\sigma = \frac{50 \times 10^3}{\frac{\pi}{4}(56^2 - 45^2)}$$

$$\sigma = 57.30 N/mm^2$$

$$\text{Strain } (e) = \frac{1.75}{2750}$$

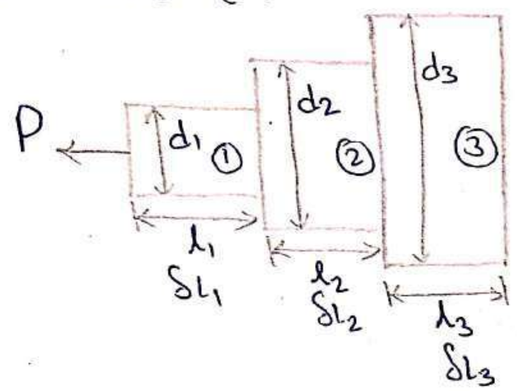
$$e = 6.36 \times 10^{-4}$$

$$E = \frac{\sigma}{e}$$

$$E = \frac{57.30}{6.36 \times 10^{-4}} \Rightarrow E = 90.09 \times 10^3 \, N/mm^2$$

## BARS OF VARYING CROSS - SECTION :- [$\delta l$]

- Main aim of the concept is to find the elongation (or) change in length.
- Let, us consider 3 bars of different sizes and cross-section as shown in the fig. which is subjected to tensile load (P).




$$\boxed{\therefore \delta l = \delta l_1 + \delta l_2 + \delta l_3}$$

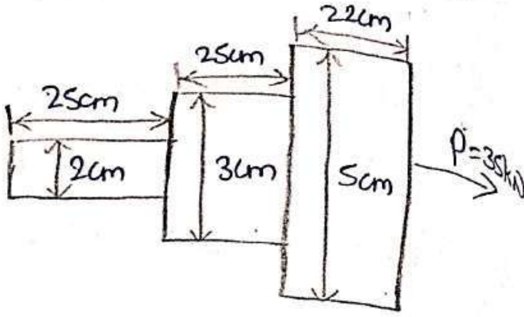
- Let, $d_1, d_2, d_3$ are the diameter of 1, 2, 3 bar respectively. $L_1, L_2, L_3$ are the length of 1, 2, 3 bar respectively.

$$E = \frac{\sigma}{\epsilon} = \frac{P}{A} \times \frac{l}{\delta l}$$

$$E = \frac{Pl}{A\delta l} \Rightarrow \delta l = \frac{PL}{AE}$$

$$\delta l_1 = \frac{PL_1}{A_1E}, \delta l_2 = \frac{Pl_2}{A_2E}, \delta l_3 = \frac{Pl_3}{A_3E}$$

Q) An Axial pull of 35,000N is acting on a bar Comprising of 3 Sub-bars as Shown in the fig. The young's Modulus is equal to $2.1\times10^5 N/mm^2$. Determine i) Stresses in each Section. ii) Total Extension of the bar.




Sol G.D :-

$P = 35\times10^3 N$ ; $E = 2.1\times10^5 N/mm^2$

$L_1 = 250mm$ ; $L_2 = 250mm$ ; $L_3 = 220mm$

$d_1 = 20mm$ ; $d_2 = 30mm$ ; $d_3 = 50mm$

i) $\text{Stress}(\sigma_1) = \dfrac{35\times10^3}{\frac{\pi}{4}(20)^2} = 111.40 N/mm^2$

ii) $\text{Stress}(\sigma_2) = \dfrac{35\times10^3}{\frac{\pi}{4}(30)^2} = 49.51 N/mm^2$

iii) $\text{Stress}(\sigma_3) = \dfrac{35\times10^3}{\frac{\pi}{4}(50)^2} = 17.82 N/mm^2$

Overall Stress of a bar $(\sigma) = 111.40 + 49.51 + 17.82$

$\sigma = 178.73 N/mm^2$

i) $\delta L_1 = \dfrac{PL_1}{A_1E} = \dfrac{35\times10^3\times250}{\frac{\pi}{4}(20)^2\times2.1\times10^5} = 0.13mm$

ii) $\delta L_2 = \dfrac{PL_2}{A_2E} = \dfrac{35\times10^3\times250}{\frac{\pi}{4}(30)^2\times2.1\times10^5} = 0.05mm$

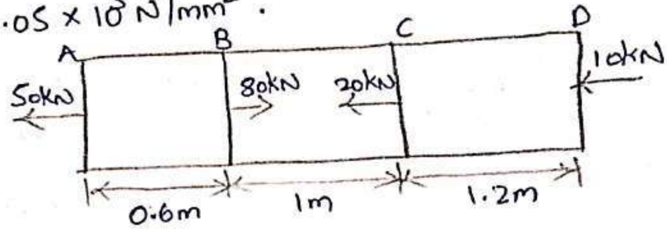
iii) $\delta L_3 = \dfrac{PL_3}{A_3E} = \dfrac{35\times10^3\times220}{\frac{\pi}{4}(50)^2\times2.1\times10^5} = 0.01 mm$

Overall elongation of a bar $(\delta L) = 0.13 + 0.05 + 0.01$

$\delta L = 0.19mm$

Q) PRINCIPLE OF SUPER POSITION :-

Q) A brass bar having Cross - Sectional area of $1000 mm^2$ is Subjected to axial forces as Shown in the fig. Find total elongation of the bar. Take $E = 1.05 \times 10^5 N/mm^2$.




Sol G.D :- $E = 1.05 \times 10^5 N/mm^2$ ; $A = 1000 mm^2$

i) AB :- 50kN 50kN

ii) BC :- 30kN $(-50+50) = -30kN$ 30kN

iii) CD :- 10kN 10kN

We know that $(\delta l) = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

$$\delta l_{\cancel{AB}} = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$$

$$= \frac{P_{AB} l_{AB}}{AE} + \frac{P_{BC} * l_{BC}}{AE} + \frac{P_{CD} * l_{CD}}{AE}$$

$$= \frac{1}{AE}\left(P_{AB} l_{AB} + P_{BC} l_{BC} + P_{CD} l_{CD}\right)$$

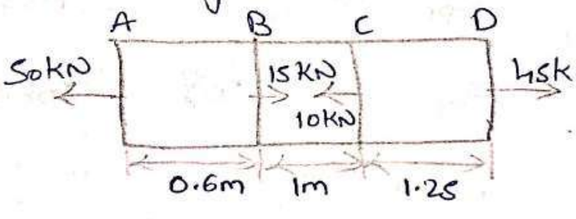
$$= \frac{1}{1000 \times 1.05 \times 10^5}\left((50 \times 10^3 \times 600) + (-(20 \times 10^3 \times 10^3) + (-(10 \times 10^3) \times 1200)\right)$$

$$= \frac{50 \times 10^3 \times 600}{1000 \times 1.05 \times 10^5} + \left(-\frac{30 \times 10^3 \times 10^3}{1000 \times 1.05 \times 10^5}\right) + \left(-\frac{10 \times 10^3 \times 1200}{1000 \times 1.05 \times 10^5}\right)$$

$$= 0.28 - \overset{0.285}{\cancel{0.19}} - 0.114$$

$$= \cancel{-0.02} - 0.114$$

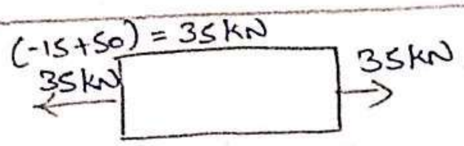
Q) A Steel bar of cross - Section $500 mm^2$ is acted upon by the forces Shown in fig. Determine the total elongation of the bar. Consider $E = 2 \times 10^5 N/mm^2$.




Sol G.D :- $A = 500 mm^2$; $E = 2 \times 10^5 N/mm^2$

AB :- 50kN 50kN

ii) BC :- $(-15+50) = 35\,kN$




iii) CD :-


$$\delta l_{AB} = \frac{Pl_{AB}}{AE} = \frac{50\times10^3\times600}{500\times2\times10^5}$$

$$= 0.3\,mm$$

$$\delta l_{BC} = \frac{Pl_{BC}}{AE} = \frac{35\times10^3\times10^3}{500\times2\times10^5}$$

$$= 0.35\,mm$$

$$\delta l_{CD} = \frac{Pl_{CD}}{AE} = \frac{45\times10^3\times1250}{500\times2\times10^5}$$

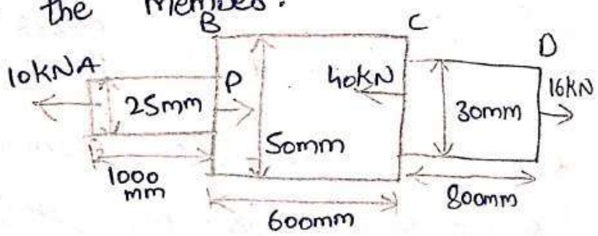
$$= 0.56\,mm$$

$$\delta L = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$$

$$= 0.3 + 0.35 + 0.56$$

$$= 1.21\,mm$$

Q) A member ABCD is Subjected to Point loads as shown below Calculate the Force (P) Necessary for equilibrium. Take $E = 2.05\times10^5\,N/mm^2$ Determine the total elongation of the member.




Sol G.D :- $E = 2.05\times10^5\,N/mm^2$

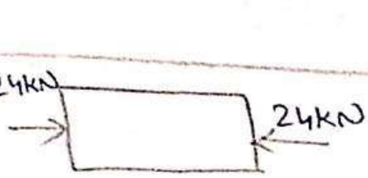
$$A_{AB} = \frac{\pi}{4}(25)^2 = 490.87\,mm^2$$

$$A_{BC} = \frac{\pi}{4}(50)^2 = 1963.4\,mm^2$$

$$A_{CD} = \frac{\pi}{4}(30)^2 = 706.85\,mm^2$$

i) AB

ii, BC




iii, CD


$$\delta l_{AB} = \frac{P l_{AB}}{AE} = \frac{10\times10^3 \times 10^3}{490.87\times 2.05\times10^5}$$

$$= 0.099mm$$

$$\delta l_{BC} = \frac{P l_{BC}}{AE} = \frac{24\times10^3\times 600}{1963.4\times2.05\times10^5}$$

$$= -0.035mm$$

$$\delta l_{CD} = \frac{P l_{CD}}{AE} = \frac{16\times10^3\times800}{706.85\times2.05\times10^5}$$

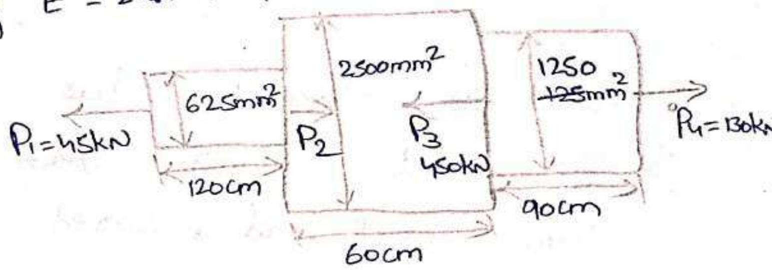
$$= 0.088mm$$

$$\delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$$

$$= \overset{0.099}{\cancel{0.094}} - 0.035 + 0.088$$

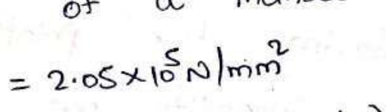
$$= 0.152mm$$

Q, A member ABCD is subjected to point loads $P_1$, $P_2$, $P_3$ and $P_4$ as shown in fig. Calculated the force $P_2$ neccessary for the equlibrium. If $P_1 = 45KN$, $P_3 = 450kN$, $P_4 = 130KN$. Determine the total elongation of a member, assuming $E = 2.\cancel{1}\times10^5 N/mm^2$.

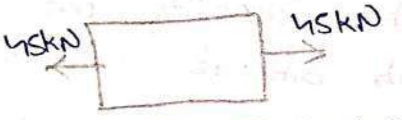



Sol G.D:- $E = 2.05\times10^5 N/mm^2$

i, AB




ii, BC




iii, CD

$$\delta l_{AB} = \frac{PL_{AB}}{AE} = \frac{45\times10^3\times1200}{625\times2.1\times10^5}$$

$$= 0.411mm$$

$$\delta l_{BC} = \frac{PL_{BC}}{AE} = \frac{320\times10^3\times600}{2500\times2.1\times10^5}$$

$$= 0.365mm$$

$$\delta l_{CD} = \frac{PL_{CD}}{AE} = \frac{130\times10^3\times900}{1250\times2.1\times10^5}$$

$$= \cancel{4.457mm}\ 0.445mm$$

$$\delta L = \delta l_{AB} - \delta l_{BC} + \delta l_{CD}$$

$$= 0.411 - 0.365 + \cancel{4.457}\ 0.445mm$$

$$= \cancel{4.503mm}\ 0.491mm$$

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## Analysis of Bars of Composite Sections :-

- A bar made of two (or) more bars of equal lengths but, of different materials rigidly fixied with ~~eq~~ each other and behaving as single unit ~~per~~ for extention (or) compression when subjected to an axial load, tensile (or) compressive is called as composite bar.
- For the composite bar the following two points are important.

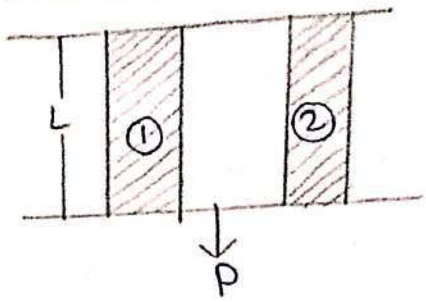
1) The total external loads on the composite bar is equal to the sum of the loads, carried by each different material's.

i.e., $P = P_1 + P_2$

2) The extension (or) comprission in each bar is equal to the sum of the loads. Hence, deformation per unit length is assumed to be same.

i.e., Strain in each bar is equal.

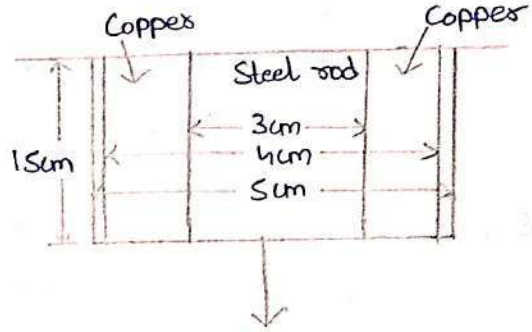
$$\delta l_1 = \delta l_2$$




Q) A Steel rod of 3cm diameter is enclosed Centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm. The composite bar is then subjected to an axial pull of 45,000 N. If the length of each bar is equal to 15cm, Determine i, the Stress in the rod and tube. ii, Load carried by each bar. Take $E_{steel} = 2.1\times10^5 N/mm^2$ and $E_{copper} = 1.1\times10^5 N/mm^2$

Sol <u>G.D:-</u> $P = 45{,}000N$ ; $E_{steel} = 2.1\times10^5 N/mm^2$ ; $E_{copper} = 1.1\times10^5 N/mm^2$

Steel dia (d) = 3cm ; Copper Inner dia = 4cm ; Copper External dia = 5cm.




$$P = P_1 + P_2$$

W.K.T $\sigma = \frac{P}{A}$

$$P = \sigma \times A$$

$$\sigma \times A = \sigma_s \times A_s + \sigma_c \times A_c$$

$$45000 = \sigma_s \times 706.85 + \sigma_c \times 706.85$$

$$\sigma_s + \sigma_c = 63.66 N/mm^2 \rightarrow (1)$$

$$e_1 = e_2$$

$$\frac{\delta l_1}{l_1} = \frac{\delta l_2}{l_2}$$

$$\therefore l_1 = l_2 = 15cm$$

$$\boxed{\delta l_1 = \delta l_2}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_{cu}}{E_{cu}} \Rightarrow \frac{\sigma_s}{\sigma_{cu}} = \frac{2.1\times10^5}{1.1\times10^5}$$

$\sigma_s = 1.90\ \sigma_{cu} \rightarrow (2)$

from eq (1) $\quad \sigma_s + \sigma_{cu} = 63.66$

$\sigma_{cu} = 21.95\ N/mm^2$

$\sigma_s = 41.75\ N/mm^2$

$$\sigma_{cu} = \frac{P_{cu}}{A_{cu}}$$

$P_{cu} = \sigma_{cu} \times A_{cu}$

$P_{cu} = 21.95 \times 706.85$

$\boxed{P_{cu} = 15.51\ KN}$

$$\sigma_s = \frac{P_s}{A_s}$$

$P_s = \sigma_s \times A_s$

$P_s = 41.75 \times 706.85$

$\boxed{P_s = 29.51\ KN}$

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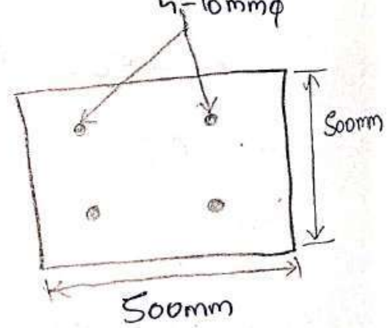
Q, A Load of 2MN is applied on a Shaft Concrete Coulmn 500mm × 500mm. The Column is reinforced with 4 Steel bars of 10mm dia. 1 inch each Corner find the Stresses in the Concrete and Steel bars. Take E steel as $2.1\times10^5\ N/mm^2$ and for concrete as $1.4\times10^4\ N/mm^2$.




Sol Given data :- Load (P) = $2\times10^6 N$

Column = 500mm × 500mm

$E_{steel} = 2.1\times10^5\ N/mm^2$

$E_{concrete} = 1.4\times10^4\ N/mm^2$

$\sigma_{st} = ?\ ;\ \sigma_{con} = ?$

i) $P = P_{st} + P_{con}$

$= \sigma_{st} \times A_{st} + \sigma_{con} \times A_{con}$

here ; $A_{st} = \cancel{4} \frac{\pi}{\cancel{4}} \times 10^2 = 314.16\,mm^2$

Total Area $= 500 \times 500 = 25 \times 10^4\,mm^2$

$A_{con} = 25 \times 10^4 - 314.16$

$= 24.90 \times 10^4\,mm^2$

$2 \times 10^6 = \sigma_{st} \times 314.16 + \sigma_{con} \times 24.96 \times 10^4 \longrightarrow ①$

ii) $\delta l_{steel} = \delta l_{concrete}$

$$\frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{con}}{E_{con}}$$

$$\sigma_{st} = \frac{\sigma_{con} \times 2.1 \times 10^5}{1.4 \times 10^4}$$

$\sigma_{st} = 15\,\sigma_{con}$ ($\because$ substitute in eq①)

$2 \times 10^6 = \sigma_{st} \times 314.16 + \sigma_{con} \times 24.96 \times 10^4$

$2 \times 10^6 = (15\sigma_{con}) \times 314.16 + \sigma_{con} \times 24.96 \times 10^4$

$\boxed{\sigma_{con} = 7.864\,N/mm^2}$ ; $\sigma_{st} = 15 \times 7.864$

$\boxed{\sigma_{st} = 117.96\,N/mm^2}$

Q) A Steel rod and 2 Copper rod together Support a load of 370kN as shown in the fig the Cross - Sectional area of steel rod is 2500mm² and of each Copper rod is 1600mm². Find the stresses in the rods take E for steel as $2 \times 10^5\,N/mm^2$ and for copper $1 \times 10^5\,N/mm^2$.

Sol Given data:- Load $(P) = 370\times10^3 N$

$A_{st} = 2500 mm^2$

$A_{cu} = 1600 mm^2$

$E_{st} = 2\times10^5 N/mm^2$

$E_{cu} = 1\times10^5 N/mm^2$

i) $P = P_1 + P_2$

$= \sigma_{st} A_{st} + \sigma_{cu} A_{cu}$

$= \sigma_{st} \times 2500 + \sigma_{cu} (1600\times2) \rightarrow eq① $

ii) $\delta l_{st} = \delta l_{cu}$

$$\frac{P l_{st}}{A_{st} E_{st}} = \frac{P l_{cu}}{A_{cu} E_{cu}} \Rightarrow \frac{\sigma_{st} l_{st}}{E_{st}} = \frac{\sigma_{cu} l_{cu}}{E_{cu}}$$

$$\boxed{\frac{370\times10^3\times150}{2500\times2\times10^5}} = \frac{\sigma_{st}\times250}{2\times10^5} = \frac{\sigma_{cu}\times150}{1\times10^5}$$

$\sigma_{st} = 1.2\sigma_{cu} \rightarrow ②$

Substitute eq② in eq①

$370\times10^3 = 1.2\sigma_{cu}\times2500 + \sigma_{cu}\times(1600\times2)$

$\sigma_{cu} = 59.6794 N/mm^2$

$\sigma_{st} = 1.27\times59.67794$

$\sigma_{st} = 71.612 N/mm^2$

Q) 3 bars made of copper, zinc & Aluminium are of equal length. And of cross - section $500 mm^2$, $750mm^2$ & $1000mm^2$ respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250kN, estimate the proportional of the load carried on each rod and the induced stresses. Take the value E for copper $= 1.3\times10^5 N/mm^2$, E for zinc is

$1\times10^5 N/mm^2$ & for Aluminium $E = 0.8\times10^5 N/mm^2$.

Sol Given data;

$$Cu \qquad\qquad Zn \qquad\qquad Al$$

$$A_{cu} = 500\,mm^2 \;;\quad A_{zn} = 750\,mm^2 \;;\quad A_{AL} = \overset{1000}{\cancel{750}}\,mm^2$$

$$E_{cu} = 1.3\times10^5 N/mm^2 \;,\quad E_{zn} = 1\times10^5 N/mm^2 \;;\quad E_{AL} = 0.8\times10^5 N/mm^2$$

$$P = 250\,kN$$

i) $P = P_{cu} + P_{zn} + P_{AL}$

$$= \sigma_{cu}A_{cu} + \sigma_{zn}A_{zn} + \sigma_{AL}A_{AL}$$

$$250\times10^3 = \sigma_{cu}\times500 + \sigma_{zn}\times750 + \sigma_{AL}\times1000 \rightarrow eq①$$

ii) $\delta l_{cu} = \delta l_{zn} = \delta l_{AL}$

$$\frac{\sigma_{cu}}{E_{cu}} = \frac{\sigma_{zn}}{E_{zn}} = \frac{\sigma_{AL}}{E_{AL}}$$

① ②

$$\frac{\sigma_{cu}}{1.3\times10^5} = \frac{\sigma_{zn}}{1\times10^5} \Rightarrow \sigma_{cu} = 1.3\sigma_{zn}$$

$$\underset{(1\times10^5)}{\frac{\sigma_{zn}}{E_{zn}}} = \underset{(0.8\times10^5)}{\frac{\sigma_{AL}}{E_{AL}}} \Rightarrow \sigma_{zn} = 1.25\,AL$$

(or)

$$\sigma_{AL} = 0.8\sigma_{zn}$$

$$250\times10^3 = 1.3\sigma_{zn}\times500 + \sigma_{zn}\,750 + 0.8\sigma_{zn}\times1000$$

$$\sigma_{zn} = 113.63\,N/mm^2$$

$$\sigma_{cu} = 1.3\times113.63 = 147.719\,N/mm^2$$

$$\sigma_{AL} = 0.8\times113.63 = 90.909\,N/mm^2$$

$$P_{cu} = \sigma_{cu}\times A_{cu} \Rightarrow 147.719\times500 = 73.85\times10^3 N$$

$$P_{zn} = \sigma_{zn}\times A_{zn} \Rightarrow 113.63\times750 = 85.222\times10^3 N$$

$$P_{AL} = \sigma_{AL}\times A_{AL} \Rightarrow 90.904\times1000 = 90.904\times10^3 N$$

$$P = P_{cu} + P_{zn} + P_{AL} \Rightarrow P = 249.976\times10^3 N$$

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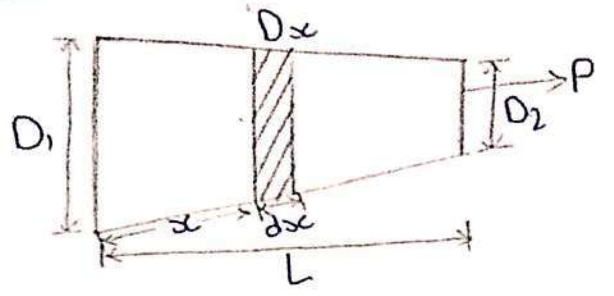
# ANALAYSIS OF CIRCULAR TAPERING ROD :-




$\delta l = ?$

$$\delta l = \frac{PL}{AE}$$

$$A_x = \frac{\pi}{4} D_x^2$$

$$D_1 = \frac{x}{L} (D_1 - D_2) = D_x$$

Inter polation Method

| (y) | (x) |
|---|---|
| $D_1$ | 0 |
| $(y_1)$ $D_x$ | $(x_1)$ $x$ |
| $(y_2)$ $D_2$ | $(x_2)$ $L$ |

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

Where, By interpolation Method

$$\frac{D_1 - D_2}{0 - x} = \frac{D_1 - D_2}{0 - L}$$

$$\left[\because D_x = D_1 - \frac{x}{L} (D_1 - D_2)\right]$$

A bar uniformly tapering from a diameter $D_1$ at one end to a diameter $D_2$ at the other end as shown in the figure. Let, $P$ = Axial tension load on the bar

$L$ = Total length of the bar.

$E$ = young's Modulus.

Consider, a small element of length $dx$ of the bar at a distance x from the left bar (or) left end.

~~Then~~

$$D_x = D_1 - \frac{x}{L} (D_1 - D_2)$$

$$D_x = D_1 - x\left[\frac{D_1 - D_2}{L}\right]$$

Assuming ; $\frac{D_1 - D_2}{L} = k$

$$D_x = D_1 - kx$$

Area of cross section of the bar at a distance $x$ from the left end.

$$A_x = \frac{\pi}{4} (D_x)^2$$

$$A_x = \frac{\pi}{4}(D_1 - kx)^2$$

Now, The Stress at a distance $x$ from the left end is given by

$$\sigma_x = \frac{P}{A_x}$$

$$\sigma_x = \frac{P}{\frac{\pi}{4}(D_1-kx)^2}$$

$$\sigma_x = \frac{4P}{\pi(D_1-kx)^2}$$

The Strain $e_x$ (or) $\epsilon_x$ in the Small element of length ~~d~~ $Dx$ is obtained by using the eq w.k.T.

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$= \frac{4P}{\pi E(D_1-\cancel{D}kx)^2}$$

Therefore, Extension of the Small elemental length ~~Dx~~

$$dx = \text{Strain} \cdot dx$$

$$= \epsilon_x \cdot dx$$

$$= \frac{4P}{\pi E(D_1-kx)^2} \cdot dx$$

Total Extension of the bar is obtained by integrating the above equation between the limits 0 & L.

Therefore, Total Extension (dL) $= \int_0^L \frac{4P}{\pi E(D_1-kx)^2} \cdot dx$

$$= \frac{4P}{\pi E}\int_0^L \frac{1}{(D_1-kx)^2} \cdot dx$$

$$= \frac{4P}{\pi E}\int_0^L (D_1-kx)^{-2} \cdot dx$$

$$= \frac{4P}{\pi E}\left[\frac{(D_1-kx)^{-2+1}}{-2+1} \times \frac{(-1)}{k}\right]_0^L$$

$$dl = \frac{4P}{\pi E}\left[\frac{(D_1 - kx)^{-1}}{\cancel{-1}} \times \frac{\cancel{-1}}{k}\right]_0^L$$

$$= \frac{4P}{\pi E}\left[\frac{1}{K(D_1 - kx)}\right]_0^L$$

$$= \frac{4P}{\pi E K}\left[\frac{1}{D_1 - L\cancel{k}} - \frac{1}{D_1 - k(0)}\right]$$

$$= \frac{4P}{\pi E K}\left[\frac{1}{D_1 - kL} - \frac{1}{D_1}\right]$$

$$= \frac{4P}{\pi E K}\left[\frac{\cancel{D_1} - (\cancel{D_1} - KL)}{D_1(D_1 - KL)}\right]$$

$$= \frac{4P}{\pi E \cancel{K}}\left[\frac{KL}{D_1(D_1 - KL)}\right]$$

$$= \frac{4P}{\pi E}\left[\frac{L}{D_1(D_1 - KL)}\right]$$

Substitute the k value,

$$= \frac{4P}{\pi E}\left[\frac{L}{D_1\left(D_1 - \left(\frac{D_1 - D_2}{\cancel{L}}\right)\cancel{L}\right)}\right]$$

$$= \frac{4P}{\pi E}\left[\frac{L}{D_1(D_2)}\right]$$

$$\boxed{\delta L = \frac{4PL}{\pi E D_1 D_2}}$$

## RELATIONSHIP BETWEEN STRESS AND STRAIN:-

$\sigma$, $\epsilon$

$$\left.\begin{array}{l}\sigma_x \rightarrow \epsilon_x \\ \sigma_y \rightarrow \epsilon_y\end{array}\right\} \quad \frac{\sigma_x}{\epsilon_x} = E \Rightarrow \frac{\sigma_x}{E} = \epsilon_x$$

$$\epsilon_y = \frac{\sigma_y}{E}$$

$$\mu = \frac{\text{lateral Strain}}{\text{longitudinal strain}} \Rightarrow e_{lat} = \mu \times e_{long}$$

$$e_{lat(x)} = \mu \times e_{x\,long}$$

$$-e_{lat} = \mu \times \frac{\sigma_x}{E}$$

$$e_{lat} = \frac{-\mu\sigma_x}{E} \longrightarrow x\text{-dir'n.}$$

$$e_{lat} = \frac{-\mu\sigma_y}{E} \rightarrow Y\text{-dir'n}$$

$\sigma_x$ :-

Strain in $x$-dir'n $= \frac{\sigma_x}{E}$

Strain in $Y$-dir'n $= -\frac{\mu\sigma_x}{E}$

$\sigma_y$ :-

Strain in $Y$-dir'n $= \frac{\sigma_y}{E}$

Strain in $x$-dir'n $= -\frac{\mu\sigma_y}{E}$

2D :- Total Strain in $x$-dir'n $= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$

" " " $Y$-dir'n $= \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$

3D :- Total Strain in $x$-dir'n $= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$

" " " $Y$-dir'n $= \frac{\sigma_y}{E} - \frac{\mu\sigma_z}{E} - \frac{\mu\sigma_x}{E}$

" " " $z$-dir'n $= \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\cancel{= \left[\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right] + \Big[}$$

$$= \left[\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}\right] + \left[\frac{\sigma_y}{E} - \frac{\mu\sigma_z}{E} - \frac{\mu\sigma_x}{E}\right] + \left[\frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}\right]$$

$$= \left[\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right] + \left[-\frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} - \frac{\mu\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}\right]$$

$$= \left[\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right] - \frac{\mu}{E}\left[2\sigma_y + 2\sigma_x + 2\sigma_z\right]$$

$$= \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z) \Rightarrow \epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

1Q) Determine the value of young's Modulus & Poision's ratio of metalic bar of length 30cm, breadth = 4cm, death 4cm. When the bar is Subjected to 400kN the decrease in length given 0.075cm and increase in breadth is 0.003 cm.

2Q) A Steel bar 300mm long, 500mm wide & 40mm thick is Subjected to a pull of 300kN in the direction of it's length. Determine the change in volume. Take $E = 2\times10^5 N/mm^2$, $\mu =$ ~~0.5~~ 0.25

12/9/25

1Sol Given that, Length = 300mm ; breadth = 40mm ; depth = 40mm

Load (P) = $400\times10^3 N$ ; $\delta l = 0.075cm = 0.75mm$ ; $\delta b = 0.003cm = 0.03mm$

i) Young's Modulus (E) $= \frac{\sigma}{\epsilon}$

$$= \frac{\frac{400\times10^3}{40\times40}}{\frac{0.75}{300}} = \frac{250}{0.0025}$$

$$= 1\times10^5 N/mm^2$$

ii) $\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

$$\mu = \frac{\frac{0.03}{40}}{\frac{0.75}{300}}$$

$$\mu = \frac{0.0025}{0.0008} \Rightarrow \mu = 0.32$$

2 Sol Given that, $L = 300mm$ ; $b = 500mm$ ; $t = 40mm$

$P = 300\times10^3 N$ ; $E = 2\times10^5 N/mm^2$ ; $\mu = 0.25$

$\delta v = ?$

$$e_v = \frac{\delta v}{V}$$

$$\delta v = e_v \times V$$

$$V = 300 \times 500 \times 40$$

$$V = 6\times10^6 mm^3$$

$$e_v = \frac{\sigma}{E}(1-2\mu)$$

$$e_v = \frac{\frac{300\times10^3}{500\times40}}{2\times10^5}(1-2\times0.25)$$

$$e_v = \frac{15}{2\times10^5}(1-2\times0.25)$$

$$e_v = 3.75\times10^{-5}$$

$$\delta v = 3.75\times10^{-5}\times6\times10^6$$

$$\delta v = 225 mm^3$$

*** Thermal Stresses :-

Case - (i) Free to Expand



$$\boxed{\sigma_T = 0}$$

$$e_T = \alpha\Delta T$$

$$\frac{\delta l}{l} = \alpha t$$

$$\boxed{\delta l = l\alpha t}$$

Case - (ii) Bar is prevented from Expansion.



Free Expansion = expansion prevented

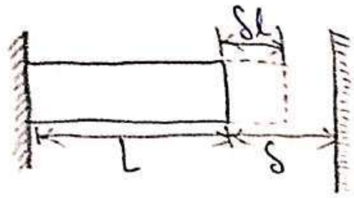
$$\delta l = \delta l$$

$$\cancel{L}\alpha t = \left[\frac{P\cancel{L}}{AE}\right] \rightarrow \sigma_T$$

$$\boxed{\sigma_T = E\alpha t}$$

Case - iii) Yielding of Supports

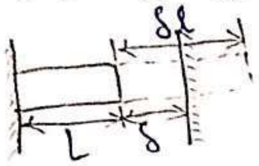
i) $\delta > \delta l$



$\sigma_T = 0$

$e_T = l\alpha T$

ii) $\delta < \delta l$



Free Expansion required = Expansion prevented

$$\delta l - \delta = \frac{PL}{AE}$$

$$E(\delta l - \delta) = \sigma_T L$$

$$\sigma_T = \frac{E(\delta l - \delta)}{L}$$

W.K.T $\Rightarrow \delta l = L\alpha T$

$$\boxed{\sigma_T = \frac{E(L\alpha T - \delta)}{L}}$$

Case - iv) Un-yielding of Support's

Same as case-ii,

$$\boxed{\therefore \alpha_T = E\alpha T}$$

$\boxed{\alpha_{cu} = 18\times10^{-6}}$ $\boxed{\alpha_{St} = 12\times10^{-6}}$ (Standard Value)

Q) A Steel rod of 3cm diameter and 5m Long is connected to Two grips and the rod is maintained at the Temperature of 95°c. Determine the Stress and pull Exherted When the Temperature falls to 30°c if i) The Ends don't yield.

ii) The Ends yield by 0.12cm.

Sol Given data:- $D = 30mm$ ; Length $(L) = 5\times10^3 mm$

$T = 95°c$ ; $\delta T = 30°c$

i) Supports / end don't yeild:-

$\sigma_T = E\alpha T$

$\sigma_T = 2\times10^5 \times 12\times10^{-6} \times (95° - 30°)$

$\sigma_T = 156 N/mm^2 \Rightarrow$ ~~110.26~~ $P = 110.26 kN$

ii) Ends yield by 0.12 cm

$\delta = 0.12\,cm = 1.2\,mm$

$\delta l = l \alpha T$

$\delta l = 5\times10^{3} \times 12\times10^{-6} \times (95^{\circ} - 30^{\circ})$

$\delta l = 3.9\,mm \quad (\because \delta > \delta l)$

$$\sigma_T = \frac{E(\delta l - \delta)}{L}$$

$\sigma_T = 108\,N/mm^2$

$$\sigma_T = \frac{P}{A}$$

$P = \sigma_T \times A$

$P = 108 \times \frac{\pi}{4}(30^2)$

$P = 76.34\,KN$

Q) A Steel rod is 25m long at a temperature of $20^{\circ}c$. Find the free ~~Expre~~ Expansion of the rod. When the temperature rise to $65^{\circ}c$ find the temperature Stresses produced

i) When the Expansion of the rod is prevented.

ii) When the rod is allow to Expand by 4mm.

Take $\alpha = 12\times10^{-6}/^{\circ}c$ & $E = 2\times10^{5}\,N/mm^2$

Sol Given data :- $L = 25\times10^{3}\,mm$ ; $T_1 = 20^{\circ}c$ ; $T_2 = 65^{\circ}c$

i) Free Expansion of the rod.

$\delta l = L\alpha T$

$\delta l = 25\times10^{3} \times 12\times10^{-6} \times (65-20)$

$\delta l = 13.5\,mm$

ii) Expansion of the rod is prevented

$\sigma_T = E\alpha T$

$= 2\times10^5 \times 12\times10^{-6} \times (65-30)$

$= 108\,N/mm^2$

iii) The rod is allowed to Expand by 4mm.

$$\sigma_T = \frac{E(\delta L - \delta)}{L}$$

$$\sigma_T = \frac{2\times10^5\,(13.5-4)}{25\times10^3}$$

$\sigma_T = 76\,N/mm^2$

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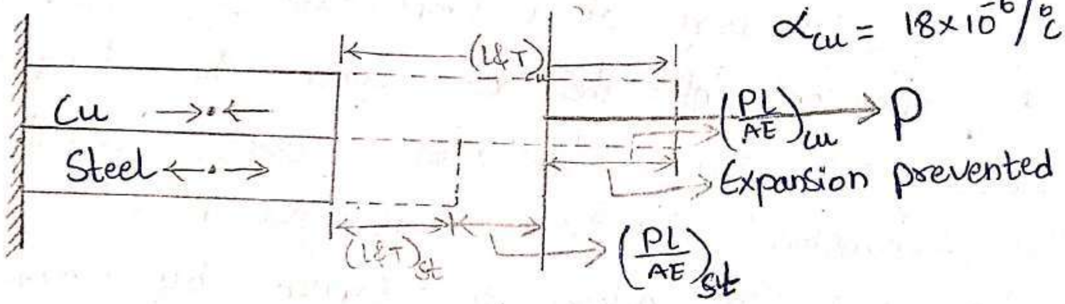
## THERMAL STRESS IN COMPOSITE BARS :-

Case - i) Temperature is increased. $(\alpha_{cu} > \alpha_{st})$

$\alpha_{cu} = 18\times10^{-6}/°C$



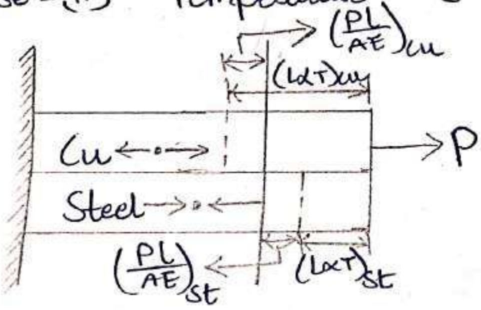

i) $P_{cu} = P_{st}$

$\sigma_{cu} \times A_{cu} = \sigma_{st} \times A_{st}$

ii) $\delta l_{cu} = \delta l_{st}$

$(L\alpha T)_{cu} - \left(\frac{PL}{AE}\right)_{cu} = (L\alpha T) + \left(\frac{PL}{AE}\right)_{st}$

Case - ii) Temperature is decreased. $(\alpha_{cu} > \alpha_{st})$



i) $P_{cu} = P_{st}$

$\sigma_{cu} \times A_{cu} = \sigma_{st} \times A_{st}$

ii) $\delta l_{cu} = \delta l_{st}$

$$(L\alpha T)_{cu} - \left(\frac{PL}{AE}\right)_{cu} = (L\alpha T)_{st} + \left(\frac{PL}{AE}\right)_{st}$$

Q) A steel tube of 30mm External diameter and 20mm internal diameter encloses a copper rod of 15mm diameter to which it is rigidly join at each end. If a temperature of 10°c there is no Longitudinal Stress. Calculate the stress in the rod and tube when temperature is rised to 200°c. Take $E_{st} = 2.1\times10^5 N/mm^2$, $E_{cu} = 1\times10^5 N/mm^2$. The value of co-efficient of linear Expansion for steel and copper is given as $11\times10^{-6}/°c$ & $18\times10^{-6}/°c$ respectively.

Sol Given data, $D_{st} = 30mm$ ; $D_{cu} = 15mm$ ; $T_2 = 200°c$

$d_{st} = 20mm$ ; $T_1 = 10°c$ ; $E_{st} = 2.1\times10^5 N/mm^2$

$E_{cu} = 1\times10^5 N/mm^2$ ; $\alpha_{cu} = 18\times10^{-6}/°c$ ; $\alpha_{st} = 11\times10^{-6}/°c$

$$A_{st} = \frac{\pi}{4}(30^2 - 20^2)$$

$$= 392.699 mm^2$$

$$A_{cu} = \frac{\pi}{4}(15^2)$$

$$= 176.714 mm^2$$

i) $P_{cu} = P_{st}$

$$\sigma_{cu} \times A_{cu} = \sigma_{st} \times A_{st}$$

$$\sigma_{cu} = \frac{\sigma_{st} \times 392.699}{176.714}$$

$$\sigma_{cu} = 2.222\sigma_{st} \rightarrow (1)$$

ii) $\delta l_{cu} = \delta l_{st}$

$$(L\alpha T)_{cu} - \left(\frac{PL}{AE}\right)_{cu} = (L\alpha T)_{st} + \left(\frac{PL}{AE}\right)_{st}$$

$$(18\times10^{-6}\times190) - \left(\frac{2.222\sigma_{st}}{1\times10^5}\right) = (11\times10^{-6}\times190) + \left(\frac{\sigma_{st}}{2\times10^5}\right)$$

$$\sigma_{st} = 49.29 N/mm^2$$

$$\sigma_{cu} = 2.222\sigma_{st}$$

$$\sigma_{cu} = 2.222 \times \cancel{109.52}\ 49.29\ N/mm^2$$

$$\sigma_{cu} = 109.522 N/mm^2$$

18/09/25

## STARIN ENERGY :-

It is the energy Stored in a body due to Strain.

Resilience :- It is the Total energy stored in a body with in the elastic limit.

Proof Resilience :- It is the maximum Strain energy Stored in a body upto elastic limit.

Modulus of Resilience :-

It is defined as the ratio of proof resilience per unit Volume.

- Strain energy principle is used in Springs.
- When the load is removed the material will regains it's Original Shape.

The load applied on a material can be

i) Gradually applied load.

ii) Suddenly applied load.

iii) Impact load (or) Load falling from a height (h).

iv)

- Strain energy is also called as work done by a loaded material.
- UNITS OF STRAIN ENERGY :-

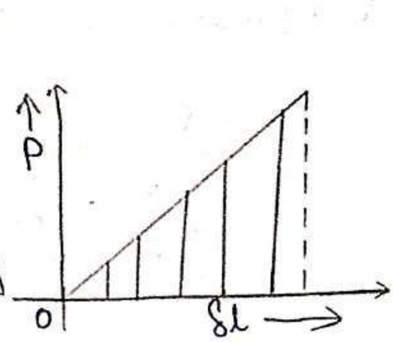
Work done = Load × distance

= N × m (or) N·m

- Strain energy depend on applied load it is denoted by (U).
- $\boxed{U = \frac{\sigma^2}{2E} \times V}$

## Case - (i) Load Applied Gradually

- Work Stored in a body must be equal to work done in a body.




Work Stored in a body = Work done in a body

$$\frac{1}{2}\times\delta l\times P \Rightarrow \frac{1}{2}\times\left(\frac{\sigma}{E}\times l\right)\times(\sigma\times A)$$

$$\Rightarrow \frac{1}{2}\times\frac{\sigma^2\times(A\times l)}{E}$$

$$\Rightarrow \frac{1}{2}\times\frac{\sigma^2}{E}\times \text{volume}$$

$$\Rightarrow \frac{\sigma^2}{2E}\times V.$$

$$\frac{1}{\cancel{2}}\times P\times\delta l = \frac{\sigma^2}{\cancel{2}E}\times(A\times l)$$

$$P\times\frac{\cancel{\sigma}}{\cancel{E}}\times\cancel{l} = \frac{\sigma^{\cancel{2}}}{\cancel{E}}\times(A\times\cancel{l})$$

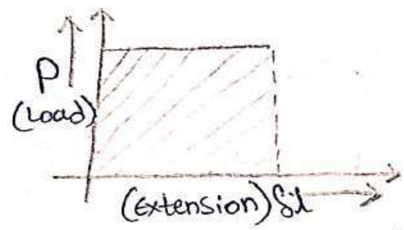
$$\cancel{P} = \cancel{\sigma} \quad P = \sigma\times A$$

$$\boxed{\sigma_g = \frac{P}{A}}$$

$$\boxed{\sigma_g = \frac{P}{A}}$$

$\sigma_g$ indicates stress corresponding to gradually applied load.

## Case - (ii) Load Applied Suddenly:-




Work stored in a body = ~~Work~~ Strain Energy produced in body.

$$P\times\delta l = \frac{\sigma^2}{2E}\times V$$

$$P\times\left(\frac{\cancel{\sigma}}{\cancel{E}}\times\cancel{l}\right) = \frac{\sigma^{\cancel{2}}}{2\cancel{E}}\times(A\times\cancel{l})$$

$$P = \frac{\sigma}{2}A$$

$$\boxed{\sigma_s = \frac{2P}{A}}$$

Q) A tensile load of 60kN is gradually applied to a circular bar of 4cm diameter & 5m long. Take $E = 2\times10^5 N/mm^2$. Determine (i) Stretch in the rod (ii) Stress in the rod.

Sol Given data; $P = 60kN$ ; $D = 4cm = 40mm$ ; $l = 5m = 5\times10^3 mm$

$E = 2\times10^5 N/mm^2$.

Gradually Applied

ii) Stress in the rod

$$\frac{P}{A} = \sigma$$

$$\sigma = \frac{60\times10^3}{\frac{\pi}{4}(40)^2}$$

$$\boxed{\sigma = 47.74\, N/mm^2}$$

i) Stretch in the rod

$$\delta l = \frac{\sigma}{E}\times l$$

$$\delta l = \frac{47.74}{2\times10^5}\times5\times10^3$$

$$\boxed{\delta l = 1.193mm}$$

Suddenly applied

$$\sigma_S = 2\times 47.74$$

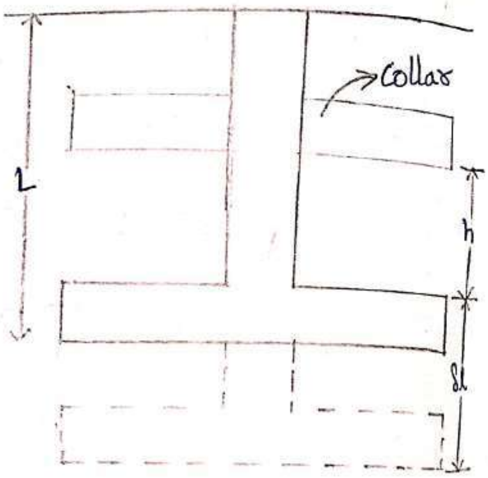
$$\boxed{\sigma_S = 95.48\, N/mm^2}$$

$$\delta l = \left(\frac{\sigma}{E}\times l\right)$$

$$\delta l = \frac{95.48}{2\times10^5}\times5\times10^3$$

$$\boxed{\delta l = 2.387mm}$$

Case - iii) Impact Load (or) load falling from a height h.




$$P\times(h+\delta l) = \frac{\sigma^2}{2E}\times(VOL)$$

$$P\times\left(h+\frac{\sigma}{E}\times L\right) = \frac{\sigma^2}{2E}\times(A\times L)$$

$$Ph + \frac{\sigma}{E}(P\times L) = \frac{\sigma^2}{2E}(A\times L)$$

$$\frac{\sigma^2}{2E}(A\times L) - \frac{\sigma}{E}(P\times L) - P\times h = 0$$

$$\sigma^2\left(\frac{AL}{2E}\right) - \sigma\left(\frac{PL}{E}\right) - P\times h = 0$$

$$\frac{AL}{2E}\left[\sigma^2 - \sigma\left(\frac{2P}{A}\right) - \frac{Ph\times2E}{AL}\right] = 0$$

$$\therefore \frac{AL}{2E}\neq 0;\quad \therefore \sigma^2 - \sigma\left(\frac{2P}{A}\right) - \frac{2PhE}{AL} = 0$$

$$(ax^2+bx+c) = 0$$

$$a=1,\quad b=\frac{-2P}{A},\quad c=\frac{-2PhE}{AL}$$

$$= \frac{\frac{2P}{A} \pm \sqrt{\left(\frac{-2P}{A}\right)^2 - 4(1)\left(-\frac{2PhE}{AL}\right)}}{2(1)}$$

$$= \frac{\frac{2P}{A} \pm \sqrt{4\left(\frac{P}{A}\right)^2 + \frac{8PhE}{AL}}}{2} = \frac{\frac{2P}{A} \pm \sqrt{4\left(\frac{P}{A}\right)^2 + \frac{8PhE}{AL}}}{2}$$

$$= \frac{\frac{2P}{A} \pm 2\sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PhE}{AL}}}{2}$$

$$= \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PhE}{AL}}$$

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Alternative Method :-

$$P \times h + \frac{\sigma}{E}(P \times L) = \frac{\sigma^2}{2E}(A \times L)$$

$$P \times h + \frac{\sigma}{E}(P \times L) = \sigma^2\left(\frac{AL}{2E}\right)$$

$$\frac{2E}{AL}\left[P \times h + \frac{\sigma}{E}(P \times L)\right] = \sigma^2$$

$$\frac{2EPh}{AL} + \sigma\left(\frac{2P}{A}\right) = \sigma^2$$

Add $\left(\frac{P}{A}\right)^2$ on both sides,

$$\frac{2EPh}{AL} + \sigma\left(\frac{2P}{A}\right) + \left(\frac{P}{A}\right)^2 = \sigma^2\left(\frac{P}{A}\right)^2 \quad (\because a^2 + b^2 - 2ab = (a-b)^2)$$

$$\frac{2EPh}{AL} + \left(\frac{P}{A}\right)^2 = \sigma^2\left(\frac{P}{A}\right)^2 - \sigma\left(\frac{2P}{A}\right)$$

$$\frac{2EPh}{AL} + \left(\frac{P}{A}\right)^2 = \sigma^2\left(\frac{P}{A}\right)^2 - 2(\sigma)\left(\frac{P}{A}\right)$$

$$\left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL} = \left(\sigma - \frac{P}{A}\right)^2$$

$$\left(\sigma - \frac{P}{A}\right) = \pm\sqrt{\left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}}$$

$$\boxed{\sigma_i = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}}}$$

Q) A load of 100N falls through a height of 2cm on to a collar rigidly attach to the lower end of a vertical bar which is 1.5m long and $1.5cm^2$ cross-section area. The upper end of the vertical bar is Determine, (i) The maximum instentaneous stress induced in the bar. (ii) The maximum instentaneous elongation. (iii) Strain energy

Sol Given data, $P = 100N$ ; $H = 20mm$ ; $l = 1.5\times10^3 mm$

$A = 1.5\times10^2 mm^2$

(i) Stress $\sigma_i = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}}$

$$\sigma_i = \frac{100}{1.5\times10^2} + \sqrt{\left(\frac{100}{1.5\times10^2}\right)^2 + \frac{2\times2\times10^5\times100\times20}{1.5\times10^2\times1.5\times10^3}}$$

$\sigma_i = 60.29 N/mm^2$

(ii) $\delta l = \frac{\sigma}{E}\times l$

$\delta l = \frac{60.29}{2\times10^5}\times1.5\times10^3$

$\delta l = 0.452mm$

(iii) Strain energy $(U) = \frac{\sigma^2}{2E}\times V$

$$U = \frac{(60.29)^2\times(1.5\times10^2\times1.5\times10^3)}{2\times2\times10^5}$$

$U = 2.04\times10^3 N.mm$

# UNIT-II

# SHEAR FORCE & BENDING MOMENT

## 1. BEAM:-

A beam is a structural element that primarly resists loads applied laterally to the axis.

### * Types of Beams:-

The following are the important types of beams

1. Cantilever beam
2. Simply Supported beam.
3. Overhanging beam.
4. Fixed beams
5. Continuous beams.

### 1.1. CANTILEVER BEAM :-

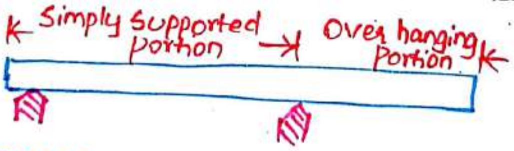
A beam which is fixed at one end and free at the other end, is known as cantilever beam.

### 1.2. SIMPLY SUPPORTED BEAM:-

A beam supported or resting on the supports freely at its both ends, is known as simply supported beam.

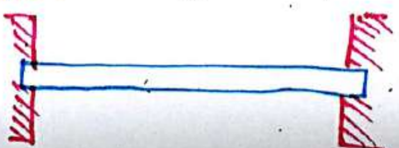
### 1.3. OVER-HANGING BEAM:-

If the end portion of a beam is extended beyond the support, such beam is known as Over hanging beam.




### 1.4. FIXED BEAMS:-

A beam whose both ends are fixed or built in walls, is known as fixed beam. A fixed beam is also known as a built in or encastred beam.



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## 1.5. Continuous Beam :-

The beam which is provided more than two supports is known as Continuous beam.

## 2. Types of Load :-

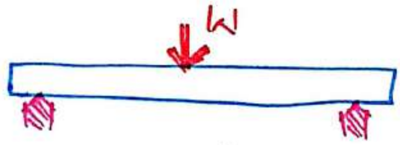
A beam is normally horizontal and the loads acting on the beams are generally vertical.

The following are the important types of load acting on a beam.

1. Concentrated or point load.
2. Uniformly distributed load.
3. Uniformly varying load.

### 2.1. Concentrated or Point load :

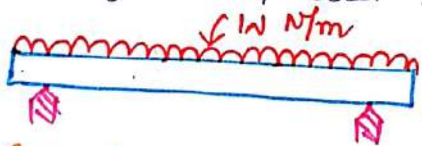
A Concentrated load is one which is considered to act at a point.




### 2.2. Uniformly distributed load :

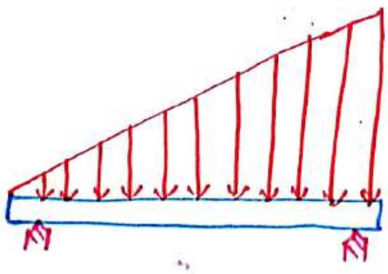
A UDL Load is one which is spread over a beam in such a manner that rate of loading 'w' is uniform along the length.

The rate of loading is expressed as "w N/m" run.




### 2.3. Uniformly Varying Load :

A UVL is one which is spread over a beam in such a manner that rate of loading varies from point to point along the beam.



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## 3. SHEAR FORCE:

Shear force at a section is the resultant of all transverse forces acting to the right or left of the section.

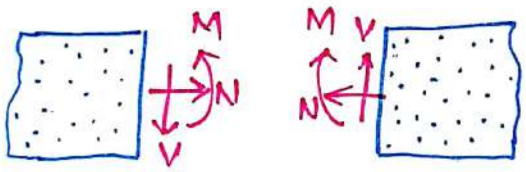
## 4. BENDING MOMENT:

Bending Moment at a section is the resultant moment at the section due to all the transverse forces either to the left or right of the section,

## 5. SIGNIFICANCE OF SFD & BMD:

At any point in a loaded member of structure when the section is cut perpendicular to the axis of the member, we get internal loading.

For Coplanar Condition i.e., when the load is in single plane, Shear force, Bending moment & Axial thrust are the internal loading.




V - Shear force at the section

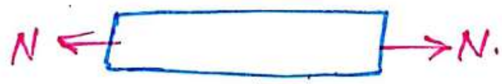
M - Bending Moment at the section.

N - Axial thrust.

## 6. AXIAL THRUST:

Axial thrust is the force acting along the longitudinal axis of the members.

Axial thrust is (+)ve if it tries to enlogate the member.

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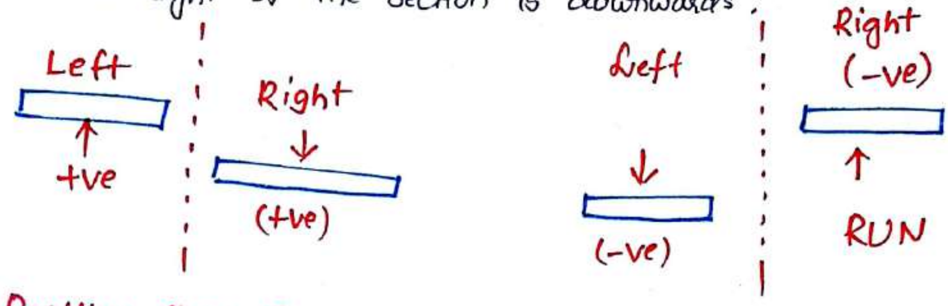
## 7. SIGN CONVENTIONS FOR SHEAR FORCE & BENDING MOMENT.

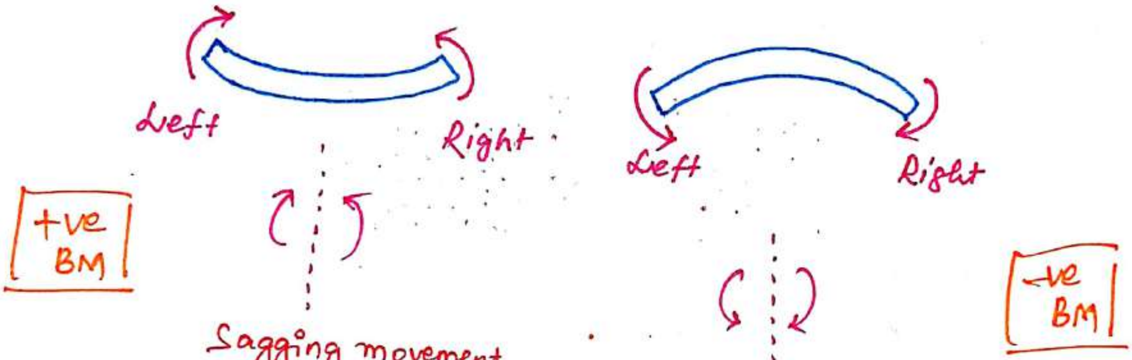
Shear Force at a section will be considered positive when the resultant of the forces to the left of the section is upwards, or the right of the section is downwards.




Positive Shear Force

Negative Shear Force.

The Bending moment will be considered positive when the moment of forces and reactions on the left portion is clockwise and on the right portion anti clock wise, vice versa.




Sagging movement

Hogging movement.

## 8. Important points for drawing SFD & BMD:.

1. Consider the left or right portion of the section
2. Add the forces normal to the beam on one of the portion
3. The positive values of SF & BM are plotted above the base line and negative values below base line
4. The SFD will increase or decrease suddenly at a section where there is a vertical point load.
5. The SF between any two vertical loads will be constant and hence the shear force diagram between two vertical loads will be horizontal.
6. The BM @ two supports of S.S. beam & free end of Cantilever is zero.

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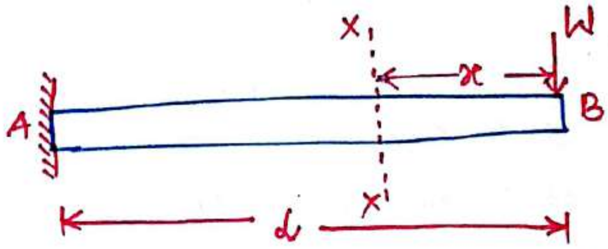
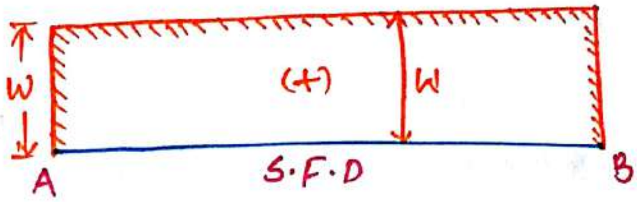
# 9. Shear Force & Bending Moment for a Cantilever with a Point Load at the Free End:

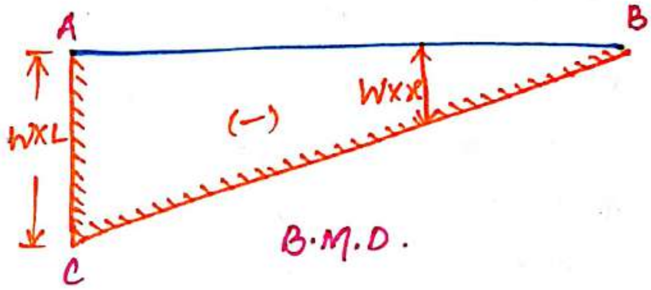



Consider a cantilever AB of length 'L' fixed at A and free at 'B' and carrying a point load 'W' at the free end 'B'.




Take a section X at a distance 'x' from the free end.

Consider the right portion of the section




The shear force at this section is resultant force acting @ right portion of given section

The resultant force acting on the right portion of the section X is 'W' and acting down wards, Hence S.F @ X is positive.

$$\boxed{F_x = +W.}$$

The shear force will be constant at all sections of the beam A & B as there is no load is present in between.

The bending moment at the section X is given by.

$$\boxed{M_x = -W \times x.}$$

Bending moment will be negative as for the right portion of the section is hogging.

at $x = 0$ i.e, @ B , BM = 0

at $x = L$ i.e., @ A , BM = $W \times L$

Hence BM follows the straight line

The shear force and Bending moment diagrams for several concentrated loads acting on a cantilever, will be drawn in the similar manner.

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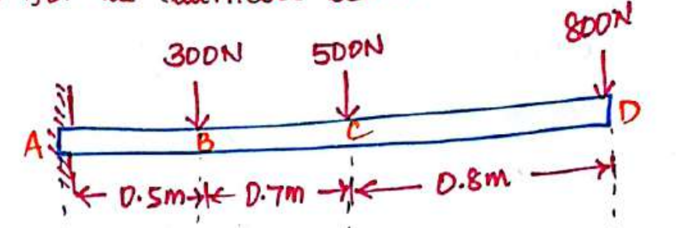
**Q:** A Cantilever beam of length 2m Carries the point loads as shown in figure. Draw the SFD & BMD for the Cantilever beam.

**Sol:**




**Shear force diagram:**

* The shear force @ D is +800N

SF remains constant b/w D & C

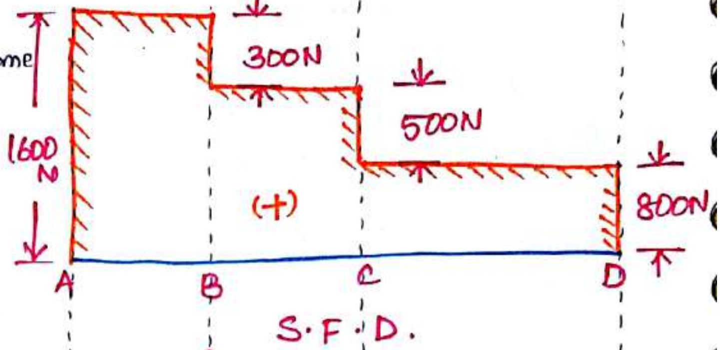
* at C due to point load SF become

$800+500 = 1300N$

* Between C & D the SF remains Constant i,e., 1300N

* @ B again SF increases to $(1300+300) = 1600N$.

* SF b/w B and A remains Constant & equals to 1600N.




SF @ D, $F_D = 800N$

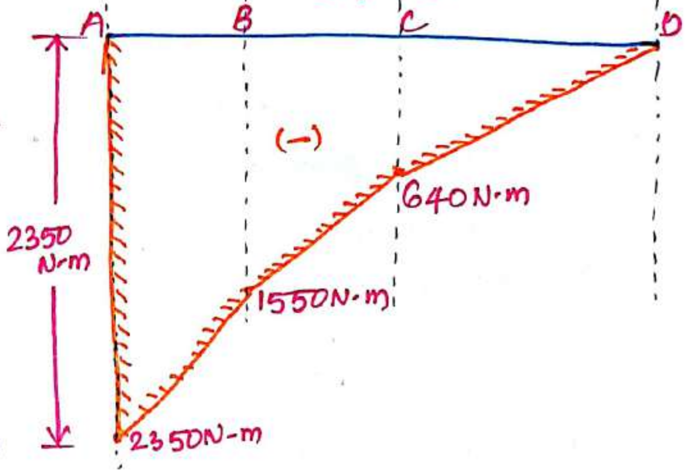
SF @ C, $F_C = +800+500$

$= 1300N$

SF @ B, $F_B = 800+500+300$

$= 1600N$.

SF @ A, $F_A = 1600N$.




**Bending Moment diagram:**

The Bending moment @ D is zero.

i, BM @ any section b/w C & D at a distance 'x' from 'D' is given by.

$$M_x = -800 \times x$$

@ C, BM $\Rightarrow M_C = -800 \times 0.8 = -640N$.

ii, BM b/w B & C at a distance 'x' from D is given by.

$$M_x = -800x - 500(x-0.8)$$

@ B is BM $\Rightarrow M_B = -800 \times 1.5 - 500(1.5-0.8)$

$= -1200 - 350 = \cancel{150}$ $-1550Nm$.

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iii, BM @ Section b/w A & B

$$M_x = -800x - 500(x-0.8) - 300(x-1.5)$$

BM @ A ;

$$-800 \times 2 - 500(2-0.8) - 300(2-1.5)$$

$$= -800 \times 2 - 500 \times 1.2 - 300 \times 0.5$$

$$= -1600 - 600 - 150 = -2350 \text{ N-m}.$$

Hence the bending moment at different points are given below as

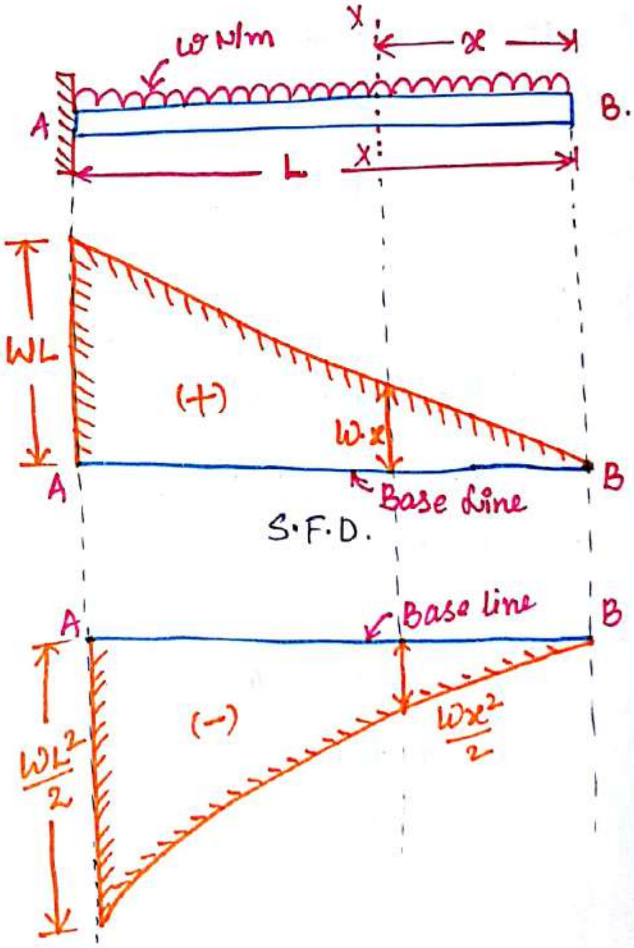
$M_D = 0$

$M_C = -640$ N-m

$M_B = -1550$ N-m

$M_A = -2350$ N-m.

## 10. SHEAR FORCE & BENDING MOMENT DIAGRAM FOR A CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD (UDL).




Consider a cantilever of length 'L' fixed at 'A' and carrying a U.D.L of 'w' per unit Length. Over the entire length of the cantilever.

The Shear force at the section 'X' will be equal to the resultant force acting on the right portion of the section.

The resultant force on the right portion = w × Length of right portion

$$\boxed{F_x = w.x}$$

The above equation shows that the shear force follows a straight line.

At B, $x = 0 \Rightarrow F_B = 0$

At A, $x = L \Rightarrow F_A = w.L$.

## Bending moment:

The U.D.L over a section is converted into point load acting at the C.G of the section.

Bending moment at the section x-x is given by.

$M_x$ = -(Total load on the right side) × Distance of C.G of right portion from $x$.

$$= -(w \cdot x) \times \frac{x}{2}.$$

$$\boxed{M_x = -\frac{wx^2}{2}.}$$

The Bending moment will be negative as for the right portion of the section, the moment of the load at '$x$' is clockwise

It is clear B.M at any section is proportional to the square of the distance of the section from the free end.

At B, $x = 0$ ; $M_x = 0$

At A, $x = L$ ; $M_x = -\frac{w \cdot L^2}{2}.$

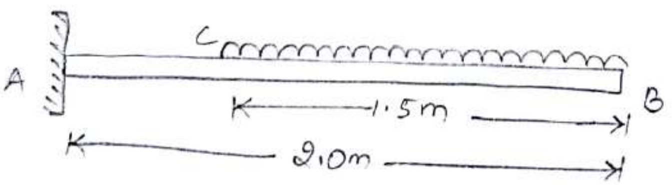
Q: A Cantilever of length 2.0m carries a uniformly distributed load of 1 KN/m run over a length of 1.5m from the free end. Draw the shear force and Bending moment diagrams for the Cantilever.

Sol:. Given

U.D.L ⇒ $w$ = 1KN/m.

Span = 2.0m

U.D.L Span = 1.5m

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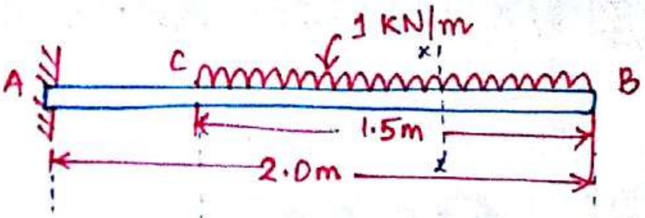
Shear force Diagram :

Consider any Section b/w C & B at a distance '$x$' from free end 'B'

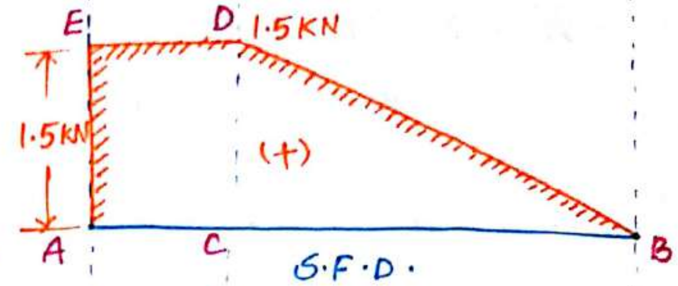



S.F @ Section $xx$

$$F_x = w \cdot x$$
$$= 1.0 \times x$$

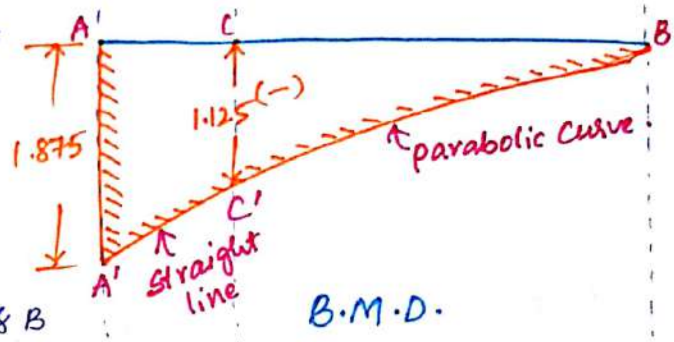
S.F @ B, $x = 0 \Rightarrow F_B = 0$

S.F @ C, $x = 1.5 \Rightarrow F_C = 1.0 \times 1.5 = 1.5\,kN$




As b/w A & C there is no load, the S.F will remain constant.

$$F_C = F_A = 1.5\,kN.$$




Bending moment diagram .

i, The B.M at any Section b/w C & B at a distance '$x$' from the free end 'B'

$$M_x = -(w \cdot x) \cdot \frac{x}{2} = -\frac{w x^2}{2}$$

B.M @ B , $x = 0 \Rightarrow M_B = 1 \times \frac{0^2}{2} = 0.$

B.M @ C , $x = 1.5 \Rightarrow M_C = 1 \times \frac{1.5^2}{2} = -1.125\,Nm$

The bending moment varies according to the parabolic law b/w C & B.

ii, The Bending moment at any Section b/w A & C at a distance '$x$' from free end 'B'

Total load due to U.D.L $= w \times 1.5 = 1.5\,kN.$

This load is acting at a distance $\frac{1.5}{2} = 0.75\,m$ from the free end.

$\therefore$ Moment b/w A & C at a distance '$x$' from free end.

$$= (\text{Load due to U.D.L}) \times (x - 0.75)$$

$$M_x = -1.5 \times (x - 0.75)$$

B.M @C , $x = 1.5\,m$ ; $M_C = -1.5(1.5 - 0.75)$

$$= -1.125\,kNm$$

B.M @A , $x = 2.0\,m$ ; $M_A = -1.5(2 - 0.75)$

$$= -1.875\,Nm.$$

B.M from C to A follows a Straight line.

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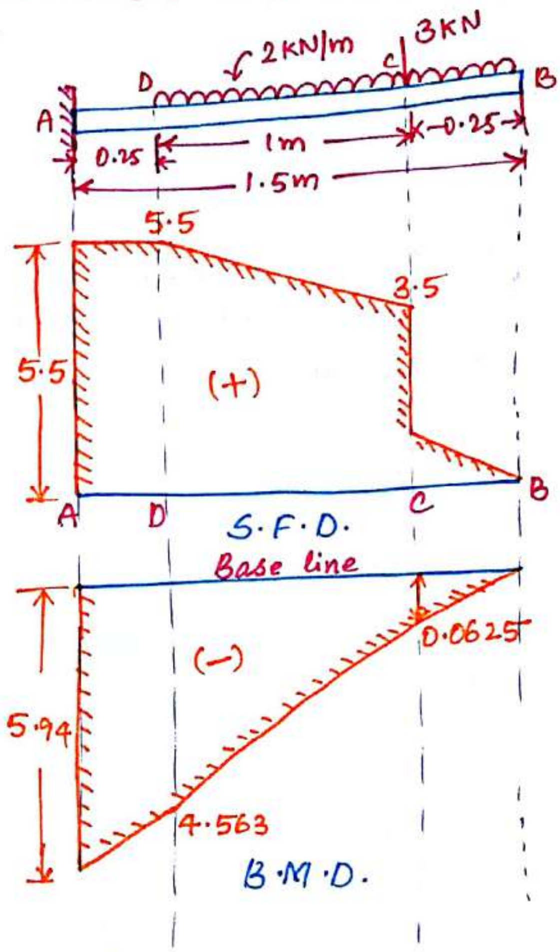
**Q:** A cantilever 1.5m long is loaded with a U.D.L of 2KN/m over a length of 1.25m from the free end. It also carries a point load of 3KN at a distance of 0.25m from free end. Draw the S.F.D & B.M.D. of the cantilever.

**Sol:**




Shear force diagram:

SF @ B ⇒ $F_B = 0$

SF @ left of C ⇒ $F_C = 2 \times 0.25$

$= 0.5\ KN$

SF @ right of C ⇒ $F_{C'} = (2 \times 0.25) + 3$

$= 3.5\ KN$.

SF b/w C & D ⇒ $F_x = w \times x + 3$

S.F @ D ⇒ $F_D = (2 \times 1.25) + 3$

$= 2.5 + 3 = 5.5\ KN$

SF from D to A follows straight line.

∴ SF @ A ⇒ $F_A = 5.5\ KN$.

Bending moment diagram:

BM @ B ⇒ $M_B = 0$.

B.M b/w B & C ⇒ $M_x = -w \cdot x \cdot \frac{x}{2} = -\frac{wx^2}{2}$

B.M @ C ⇒ $M_C = -2 \times 0.25 \times \frac{0.25}{2} = -0.0625\ kNm$

B.M b/w C & D ⇒ $M_{x'} = -w \cdot x \cdot \frac{x}{2} - 3 \cdot x$

B.M @ D ⇒ $M_D = -2 \times 1.25 \times \frac{1.25}{2} - 3 \times 1$

$= -4.563\ KNm$.

B.M @ A ⇒ $M_A = -2 \times 1.25 \times \left(\frac{1.25}{2} + 0.25\right) - 3 \times 1.25$

$= -5.94\ KN\text{-}m$.

The Bending moment b/w B & C and b/w C & D varies by a parabolic law. But the B.M b/w A & D varies by a straight line.

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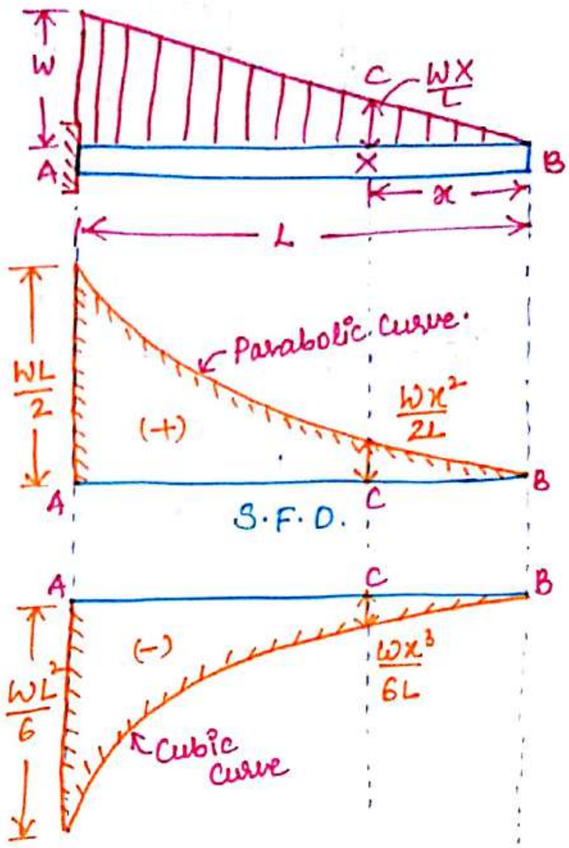
# 11. SHEAR FORCE & BENDING MOMENT DIAGRAMS FOR A CANTILEVER BEAM CARRYING U.V.L:

Consider a cantilever of length 'L' carries a gradually varying load from zero to w at fixed end.




Take a section x-x at 'x' distance from free end 'B'

The shear force at section X at a distance x from free end is given by

$F_x$ = Total load on the cantilever

= Area of triangle CBX

$= \frac{1}{2} \times (BX) \times (CX)$

$= \frac{1}{2} w \cdot \left(\frac{w \cdot x}{L}\right)$

$= \frac{w \cdot x^2}{2L}$

SF At B, $x = 0 \Rightarrow \boxed{F_B = 0.}$

SF @ A, $x = L \Rightarrow \boxed{F_A = \frac{w \cdot L}{2}.}$

$\left[\begin{array}{ll} \because \text{Loading} & 0 - 0 \\ & L - W \\ & x - ? \\ & \frac{w \cdot x}{L} \end{array}\right]$

The Bending moment at the section 'X' is

$M_x$ = −(Total load for a length x) × Distance of the load from 'X'

= −(Area of the $\Delta BCX$) × C.G of $\Delta^{le}$ from X

$= -\left[\frac{w \cdot x^2}{2L}\right] \times \left[\frac{x}{3}\right]$

$= -\frac{w \cdot x^3}{6L.}$

BM @ B, $x = 0 \Rightarrow \boxed{F_B = 0}$

BM @ A, $x = L \Rightarrow \boxed{F_A = -\frac{w \cdot L^2}{6.}}$

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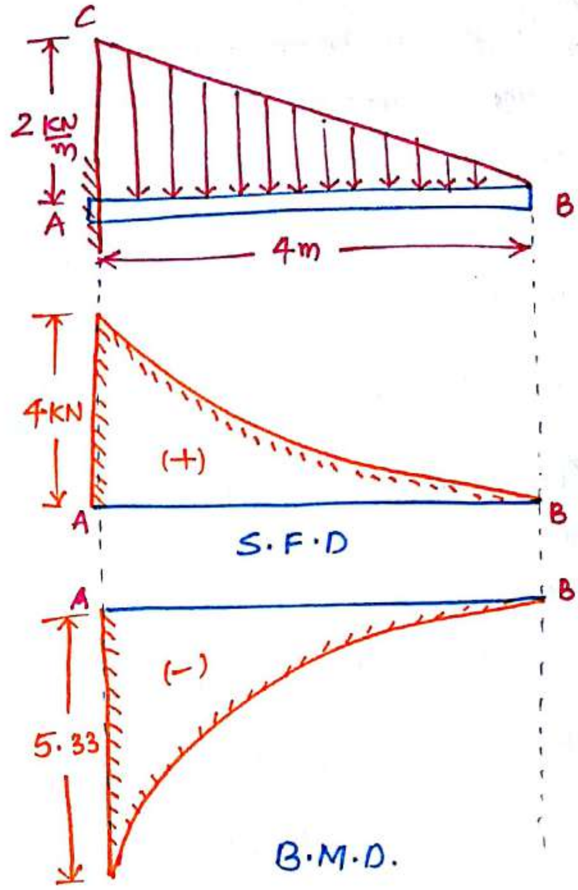
**Q**: A Cantilever of length 4 m carries a gradually varying load, zero at the free end to 2 KN/m at the fixed end. Draw the S.F.D & B.M.D for the cantilever.




**Sol:** Given

Length = 4m

$w = 2$ KN/m

## Shear force diagram

The shear force is zero @ B

SF @ A = Area of the load diagram ABC

$= \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 4 \times 2$

$= 4$ KN.

## Bending moment diagram:

BM @ B is zero

BM @ A is equal to $-\frac{w \cdot L^2}{6}$

(or) −(Area of the load diagram × Distance of C.G from A)

$= -\left(\frac{1}{2} \times AB \times AC\right) \times \frac{4}{3}$

$= -4 \times \frac{4}{3}$

$= -5.33$ KN-m.

The Bending moment between A & B varies according to Cubic Law.

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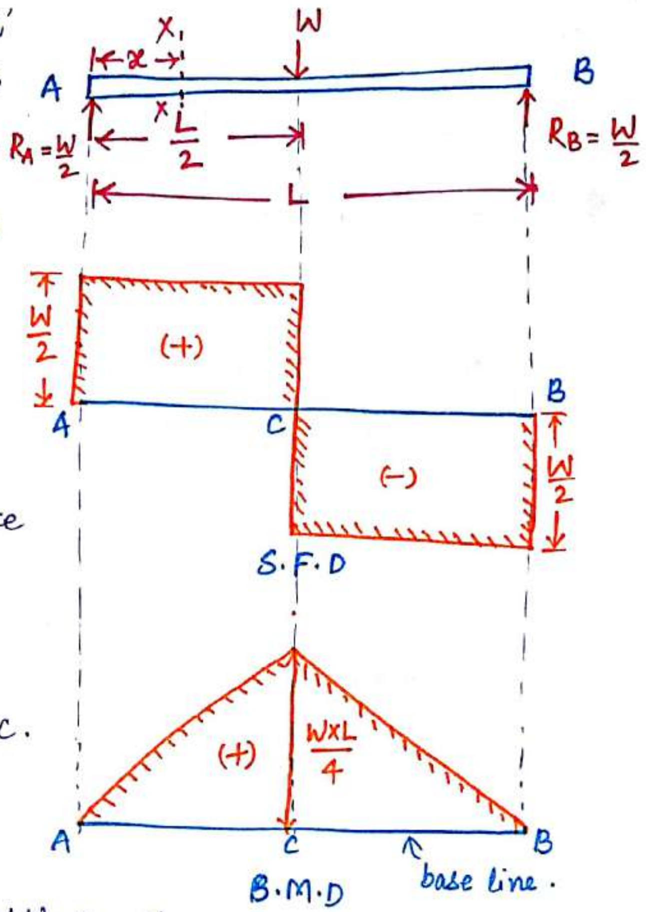
# 12. SHEAR FORCE & BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH A POINT LOAD AT MID POINT.

Consider a beam AB of length 'L' Simply Supported at the ends A & B Carrying a point load 'W' @ centre.




The reactions at the Supports will be equal to $\frac{W}{2}$

$$R_A = R_B = \frac{W}{2}.$$

Considering the right portion, the Shear force at 'x' is resultant force acting

$$F_x = -\frac{W}{2} \quad (\because RUN).$$

Now Consider any Section b/w A & C.

$$SF. = -\frac{W}{2} + W = \frac{W}{2}.$$

$\therefore$ at C shear force Changes from $+\frac{W}{2}$ to $-\frac{W}{2}$.

**Bending moment diagram:**

i, BM at any section b/w A & C at a distance 'x' from the end 'A' is given by.

$$M_x = R_A \cdot x$$

$$M_x = +\frac{W}{2} \cdot x$$

At A, $x = 0 \Rightarrow M_A = \frac{W}{2} \times 0 = 0$

At C, $x = \frac{L}{2} \Rightarrow M_C = \frac{W}{2} \times \frac{L}{2} = \frac{W \times L}{4}.$

ii, B.M at any section between C & B at a distance 'x' from end 'A'

$$M_x = R_A \cdot x - W \times \left(x - \frac{L}{2}\right)$$

At C, $x = \frac{L}{2} \Rightarrow M_C = \frac{W}{2} \times \frac{L}{2} - W\left(\cancel{\frac{L}{2} - \frac{L}{2}}\right) = \frac{WL}{4}$

At B, $x = L \Rightarrow M_B = \frac{WL}{2} - \frac{W}{2} \times L = 0,$

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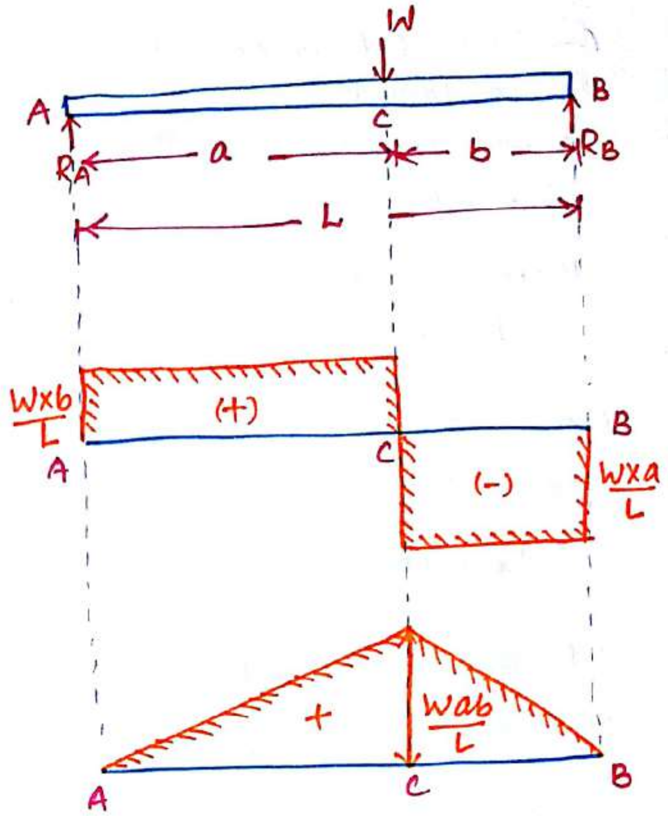
## 13. SHEAR FORCE & BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH AN ECCENTRIC POINT LOAD.




Shear force diagram.

Taking moments about A.

$$R_B \times L = W \times a$$

$$R_B = \frac{W \cdot a}{L}.$$

Illy $R_A = \frac{W \cdot b}{L}.$

S.F @ section b/w C & B

$$F_x = R_B = -\frac{W \cdot a}{L}$$

S.F @ section b/w A & C

$$F_x = \cancel{A} - R_B + W$$

$$= -\frac{W \cdot a}{L} + W$$

$$= \frac{W(L) - Wa}{L} = \frac{Wa + Wb - Wa}{L} = \frac{W \cdot b}{L}.$$

Bending moment diagram.

i, The Bending moment at any Section between A & C at a distance '$x$'

$$M_x = R_A \times x = +\frac{Wb}{L} \cdot x$$

At A, $x = 0$; $M_A = \frac{W \cdot b}{L} \times 0 = 0$

At C, $x = a$; $M_C = \frac{W \cdot b}{L} \times a = \frac{Wab}{L}.$

Hence the B.M increases from zero to $\frac{W \cdot a \cdot b}{L}$ at C by a Straight line

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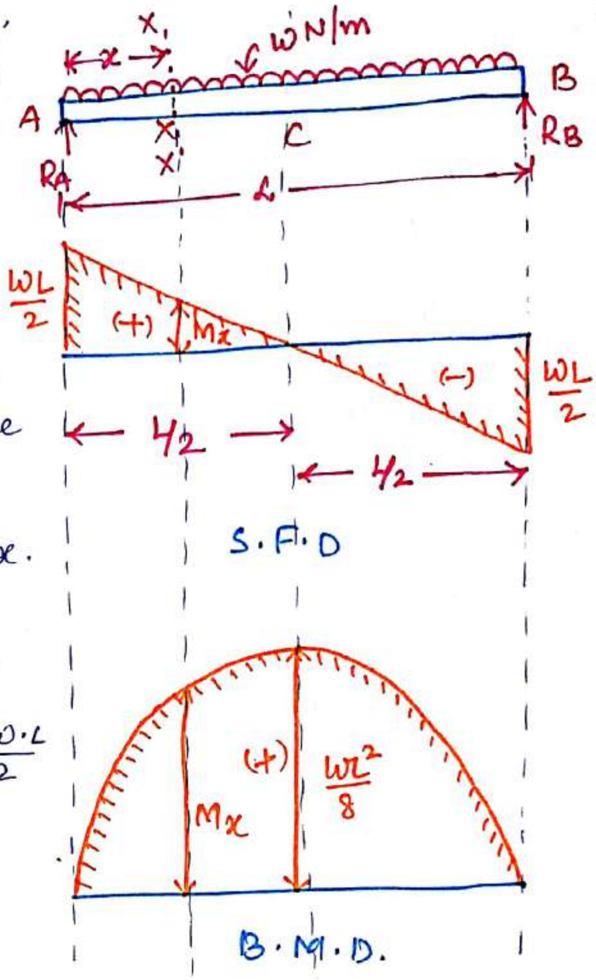
## 14. S.F.D & B.M.D FOR A SIMPLY SUPPORTED BEAM CARRYING A UNIFORMLY DISTRIBUTED LOAD:

Consider a S.S. beam AB of length 'L' with a uniformly distributed load of intensity w N/m over the entire span.




Reaction at A & B.

$$R_A = R_B = \frac{w \cdot L}{2}$$

Consider any section 'X' at a distance 'x' from the left end 'A'

$$F_x = +R_A - w \cdot x = +\frac{wL}{2} - wx.$$

S.F varies according to a st. line.

SF @ A, $x=0$ ; $F_A = +\frac{wL}{2} - \frac{w \times 0}{2} = +\frac{w \cdot L}{2}$

SF @ B ; $x = L$ ; $F_B = +\frac{wL}{2} - wL = -\frac{w \cdot L}{2}$

SF @ C, $x = \frac{L}{2}$ , $F_C = +\frac{wL}{2} - \frac{wL}{2} = 0$.

The bending moment at the section 'X' at a distance 'x' from left end 'A' is given by

$$M_x = +R_A \cdot x - w \cdot x \cdot \frac{x}{2}$$

$$= \frac{wL}{2} \cdot x - \frac{w \cdot x^2}{2}$$

B.M varies according to parabolic law.

At A, $x = 0$, $M_A = \frac{w \cdot L}{2} \cdot 0 - \frac{w \cdot 0}{2} = 0$

At B, $x = L$, $M_B = \frac{wL}{2} \cdot L - \frac{w}{2} \cdot L^2 = 0$

At C, $x = \frac{L}{2}$ , $M_C = \frac{w \cdot L}{2} \cdot \frac{L}{2} - \frac{w}{2}\left[\frac{L}{2}\right]^2$

$$= \frac{w \cdot L^2}{4} - \frac{wL^2}{8}$$

$$= +\frac{wL^2}{8}.$$

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## 15. SFD & BMD FOR A S.S.BEAM CARRYING U.V.L FROM ZERO AT ONE END TO $\omega$ per unit length AT THE OTHER END.

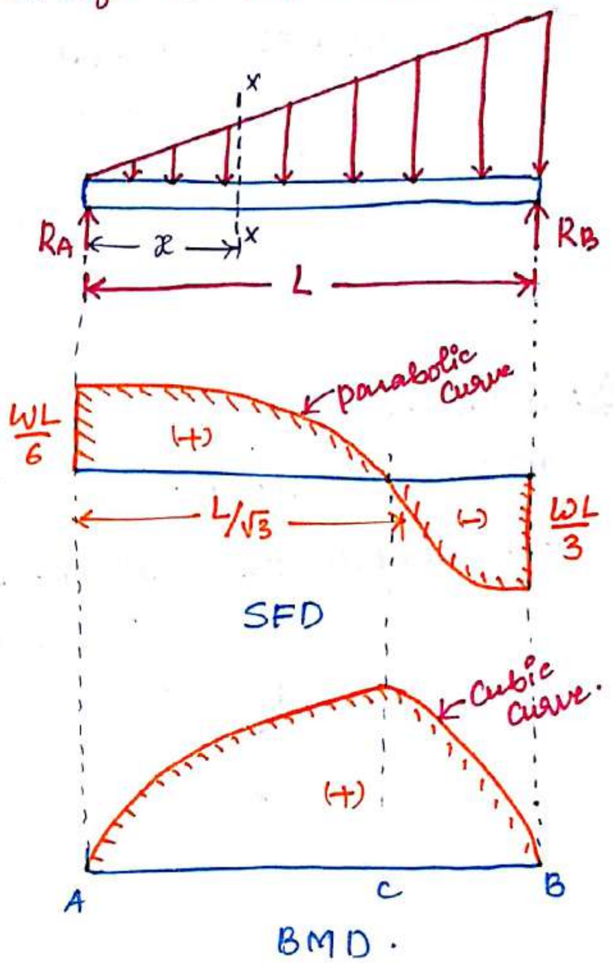
Consider a S.S beam of length 'L'

Taking moments about A, we get

$$R_B \times L = \left[\frac{\omega \cdot L}{2}\right] \times \frac{2}{3} L$$

$$R_B = \frac{\omega \cdot L}{3}$$

lly $R_A = \frac{\omega \cdot L}{6}.$




**SFD**

Consider a section $x$-$x$ at a distance '$x$' from 'A'

$$F_x = R_A - \text{Load on length } Ax$$

$$= \frac{\omega \cdot L}{6} - \frac{\omega \cdot x}{L} \cdot \frac{x}{2}$$

$$= \frac{\omega L}{6} - \frac{\omega x^2}{2L}.$$

SF @ A, $x = 0$; $F_A = \frac{\omega \cdot L}{6} - \frac{\omega}{2L} \times 0$

$$= \frac{\omega \cdot L}{6}.$$

SF @ B, $x = L$; $F_B = \frac{\omega \cdot L}{6} - \frac{\omega \cdot L^2}{2L} = -\frac{\omega \cdot L}{3}$ (SF follows a parabolic law).

SF=0 @ $\frac{\omega L}{6} - \frac{\omega x^2}{2L} = 0 \Rightarrow x^2 = \frac{\omega \cdot L}{6} \times \frac{2L}{\omega} = \frac{L^2}{3} \Rightarrow x = \frac{L}{\sqrt{3}}$

**BMD**

BM @ A & B = 0.

BM @ section $x$-$x$; $M_x = R_A x - \text{load on length } Ax \cdot \frac{x}{3}$

$$= \frac{\omega \cdot L}{6} \cdot x - \frac{\omega x^2}{2L} \cdot \frac{x}{3} = \frac{\omega L}{6} \cdot x - \frac{\omega x^3}{6L}$$

Equation shows BM varies as per Cubic law.

Maximum BM @ $L/\sqrt{3}$ $= \frac{\omega \cdot L}{6} \cdot \frac{L}{\sqrt{3}} - \frac{\omega}{6L} \cdot \left(\frac{L}{\sqrt{3}}\right)^3$

$$= \frac{\omega L^2}{6\sqrt{3}} - \frac{\omega L^2}{18\sqrt{3}}$$

$$= \frac{3\omega \cdot L^2 - \omega L^2}{18\sqrt{3}} = \frac{\omega L^2}{9\sqrt{3}}.$$

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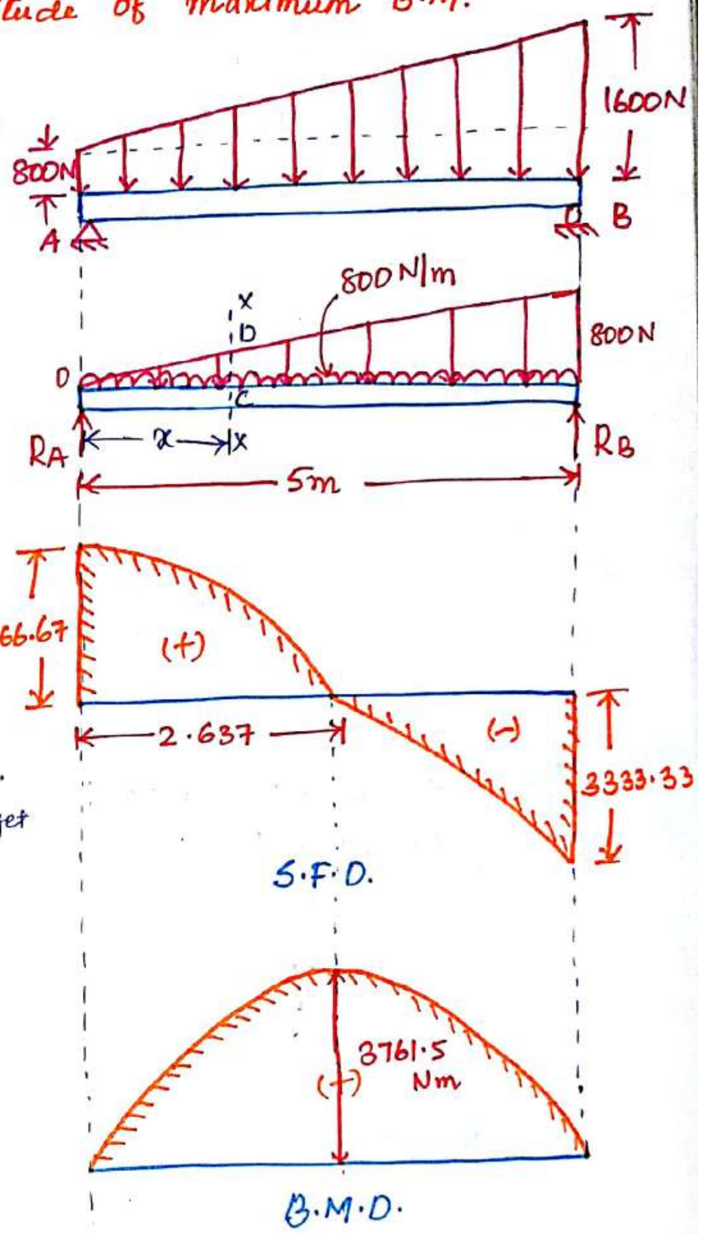
**Q**: A simply supported beam of length 5m carries a uniformly increasing load of 800 N/m run at one end to 1600 N/m run at the other end. Draw the SF & BM diagrams for the beam. Also calculate the position and magnitude of maximum B.M.

**Sol**:




The load may be assumed to be consisting of a U.D.L of 800 N/m over the entire span and a U.V.L of zero at A & 800 N/m at B.

Load on the beam due to U.D.L of 800 N/m $= 800 \times 5$

$= 4000$ N.

Load on the beam due to U.V.L

$= \frac{1}{2} \times 800 \times 5$

$= 2000$ N.

Now calculate the reactions $R_A$ & $R_B$.

Taking the moments about A, we get

$R_B \times 5 = 4000 \times \frac{5}{2} + 2000 \times \left[\frac{2}{3} \times 5\right]$

$= 3333.33$ N.

$R_A =$ Total load on beam $- R_B$

$= (4000 + 2000) - 3333.33$

$= 2666.67$ N.

Consider any section X-X at a distance '$x$' from A.

Length of CD $\Rightarrow$ $5 \to 800$, $x \to ?$ $\Rightarrow \frac{800x}{5} = 160x$.

Shear force diagram:

Now consider section X-X and shear force @ X-X

$$F_X = +R_A - (800 \times x) - \left(\frac{1}{2} \times x \times 160x\right)$$

$$\boxed{F_x = 2666.67 - \left[800x + 80x^2\right]}$$

equation follows a quadratic form, which shows it follow a parabolic path.

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SF @ A ; $2666.67 - [800 \times 0 + 80(0)^2]$

$= 2666.67\ N$

SF @ B : $2666.67 - [800 \times 5 + 80 \times 5^2]$

$= -3333.33 N$.

let us find the position of zero shear.

$$F_0 = 2666.67 - [800x + 80x^2] = 0$$

$$\Rightarrow x^2 + 10x - 33.33 = 0.$$

The above is a quadratic equation, its solution is given by

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-33.33)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{233.33}}{2} \quad \text{[neglecting -ve roots].}$$

$$= \frac{-10 + 15.274}{2}$$

$$= 2.637\ m.$$

Bending moment diagram:

BM at a section x-x is given by

$$M_x = R_A \times x - \left[\left(800x \times \frac{x}{2}\right) + \left(\frac{1}{2} \cdot x \cdot 160x \times \frac{x}{3}\right)\right]$$

$$\boxed{M_x = 2666.67x - 400x^2 - \frac{80}{3}x^3}$$

Curve follows Cubic path.

Since it is a ~~Cantilever~~ Simply Supported beam,

$$M_A = M_B = 0.$$

Bending moment at zero shear force point

$$M_0 = 2666.67(2.637) - \left[400(2.637)^2 + \frac{80}{3}(2.637)^2\right]$$

$$= \underline{3761.5\ N\text{-}m}$$

∴ Maximum B.M will occur at 2.637 m from A with a magnitude of 3761.5 N-m.

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## 16. SHEAR FORCE & BENDING MOMENT DIAGRAMS FOR OVER-HANGING BEAMS :

If the end portion of a beam is extended beyond the support, such beam is known as over hanging beam.

In case of over hanging beams, the BM is +ve between the two supports, whereas the BM is negative for the over hanging portion.

Hence, at some point, the BM is zero after changing its sign from positive to negative, vice versa.

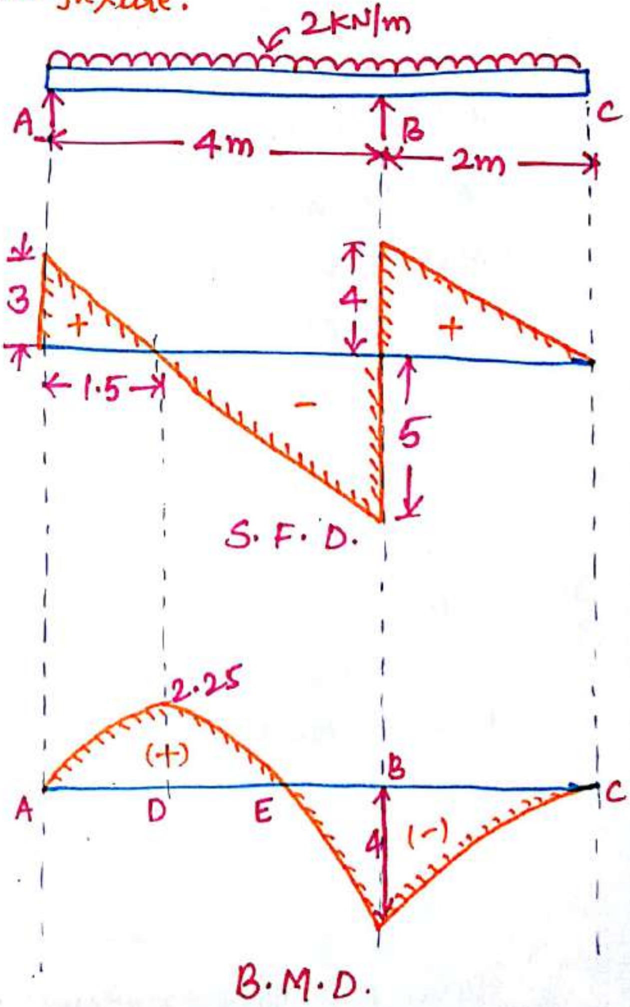
That point is known as the "Point of Contraflexure" & "point of inflexion"

## 17. POINT OF CONTRAFLEXURE :

The point where the B.M is zero after changing its sign from positive to negative & vice versa.

**Q :** Draw the shear force & Bending moment diagram for the overhanging beam carrying UDL of 2KN/m over a entire length. Also locate the point of contra flexure.

**Sol :** First calculate the reactions at $R_A$ & $R_B$.




Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times \frac{6}{2} = 36$$

$$R_B = 9\,KN$$

and

$$R_A = \text{Total load} - R_B$$

$$= 2 \times 6 - 9$$

$$= 3\,KN.$$

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Shear force diagram

$SF @ A = +R_A = +3KN.$

i, SF at any section b/w A & B at a distance 'x' from 'A'

$$F_x = R_A - 2x$$

$@A, x=0 \Rightarrow F_A = 3KN$

$@B, x=4 \Rightarrow F_B = 3-2(4) = -5KN.$

SF varies according to a st. line law b/w A & B.

The point where S.F is zero, is obtained by substituting $F_x = 0$

$$0 = 3 - 2x \Rightarrow x = \frac{3}{2} = 1.5m \text{ from A}$$

ii, SF at any section b/w B & C at a distance 'x' from A

$$F_x = +R_A - 4\times2 + R_B - (x-4)\times2$$
$$= 4 - 2(x-4).$$

At B, $x = 4m \Rightarrow F_B = 4-2(4-4) = 4KN$

At C, $x = 6m \Rightarrow F_C = 4-2(6-4) = 0.$

Bending moment diagram:-

$BM @ A = 0.$

i, BM at any section b/w A & B

$$M_x = R_A \times x - 2\times x\times\frac{x}{2} = 3x - x^2$$

At A, $x=0 \Rightarrow M_A = 0$

At B, $x=4 \Rightarrow M_B = 3\times4 - 4^2 = -4KN.$

At D, $x = 1.5m$ $M_D = 3\times1.5 - 1.5 = 4.5 - 2.25 = 2.25 KNm$

The BM b/w A & B varies according to parabolic law.

ii, BM @ any section b/w B & C at a distance 'x' is given by

$$M_x = R_A\times x - 2\times x\times\frac{x}{2} + R_B\times(x-4)$$
$$= 3x - x^2 - 9(x-4)$$

At B, $x=4 \Rightarrow M_B = 3\times4 - 4^2 + 9(4-4) = 4 KNm$

At C, $x=6 \Rightarrow M_C = 3\times6 - 6^2 + 9(6-4) = 18-36+18 = 0.$

Point of Contraflexure:

$$M_x = 3x - x^2$$

Equating $M_x$ to zero for point of contraflexure, we get

$$3x^2 - x^2 = 0 \Rightarrow x(3-x) = 0$$
$$3 - x = 0$$
$$x = 3m.$$

Hence point of contraflexure will be at 3m from A.

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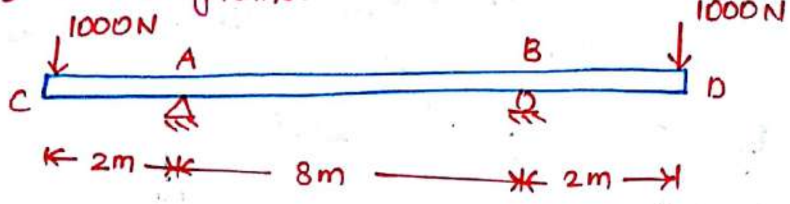
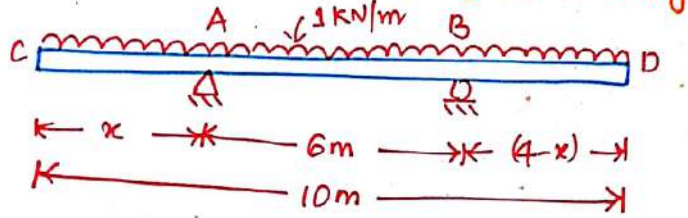
Q: A beam of length 12m is Simply Supported at two ends which are 8m apart, with an overhanging of 2m on each side as shown. The beam carries a concentrated load of 1000N at each end. Draw S.F & B.M diagrams.




Q: A horizontal beam 10m long carrying a U.D.L of 1 kN/m. The beam is Supported on two Supports 6m apart. find the position of the Supports, so that BM on The beam is as small as possible. Also draw The S.F & B.M Diagrams.




17. SHEAR FORCE & BENDING MOMENT DIAGRAMS FOR BEAMS CARRYING INCLINED LOAD:

When ever a beam carries an inclined load, they are resolved into their vertical and horizontal components

The vertical components only will cause shear force and Bending moment.

The horizontal components of the inclined loads will introduce axial force or thrust in the beam

In most of the cases, the variation of axial force for all sections of the beam can be shown by a diagram, known as thrust diagram or axial force diagram.

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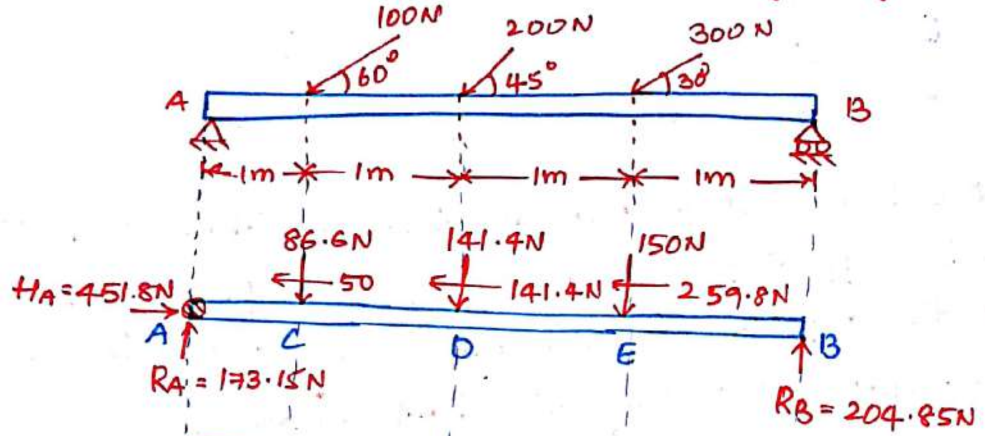
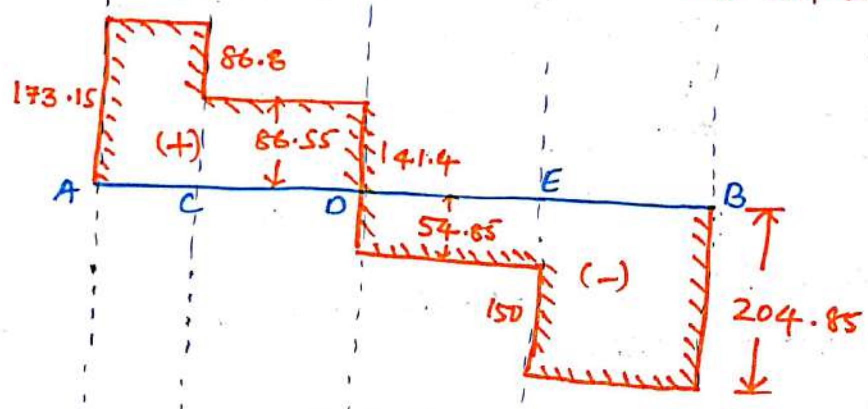
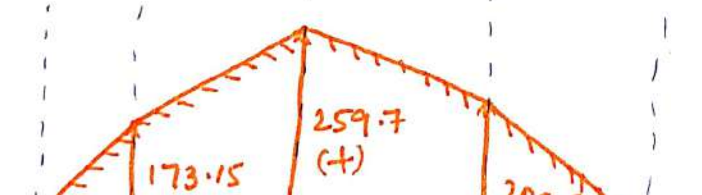
Q: A Horizontal beam AB of length 4m is hinged at 'A' and supported on roller at 'B'. The beam carries inclined loads of 100N, 200N and 300N inclined at 60°, 45° and 30° to the horizontal as shown. Draw the S.F & BM diagrams & thrust diagram for the beam.

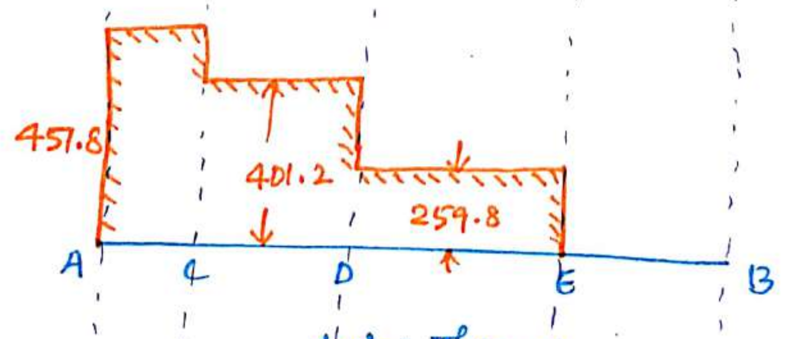



S.F.D.




B.M.D.




Axial Thrust

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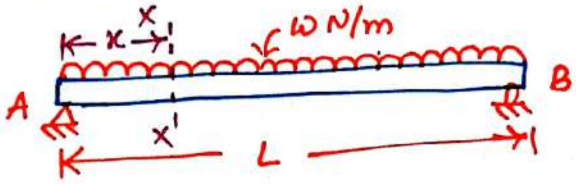
## 18. RELATION BETWEEN S.F., B.M., and RATE OF LOADING AT A SECTION OF A BEAM.

Let us consider a Simply Supported beam of length 'L' Subjected to a uniformly distributed load of 'w' N/m




Considering the moments about A & B we will get

$$R_A = R_B = \frac{wL}{2}$$

Consider a section x-x which is at a distance 'x' from Support 'A'

Shear force at Section X-X

$$\Rightarrow F_x = R_A - wx = \frac{wL}{2} - wx.$$

Bending moment about X-X

$$M_x = R_A x - wx\left(\frac{x}{2}\right)$$

$$M_x = \frac{wLx}{2} - \frac{wx^2}{2}$$

If we differentiate M with respect to 'x'

$$\frac{dM}{dx} = \frac{wL}{2} \cdot \frac{dx}{dx} - \frac{w}{2}\left(2x \cdot \frac{dx}{dx}\right)$$

$$= \frac{wL}{2} - wx = \text{Shear force.}$$

$$\Rightarrow \boxed{\frac{dM}{dx} = V.}$$

Thus, the rate of change of Bending moment with respect to 'x' is equal to the Shearing force

(or)

The slope of the moment diagram at the given point is the Shear at that point.

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Now

differential S.F, Fx with respect to 'x' gives

$$\frac{dV}{dx} = 0-\omega$$

$$\boxed{\frac{dV}{dx} = \text{Load}}$$

Thus, the rate of change of the shearing force with respect to 'x' is equal to the load intensity

(or)

The Slope of the Shear diagram at a given point equals the load at that point.

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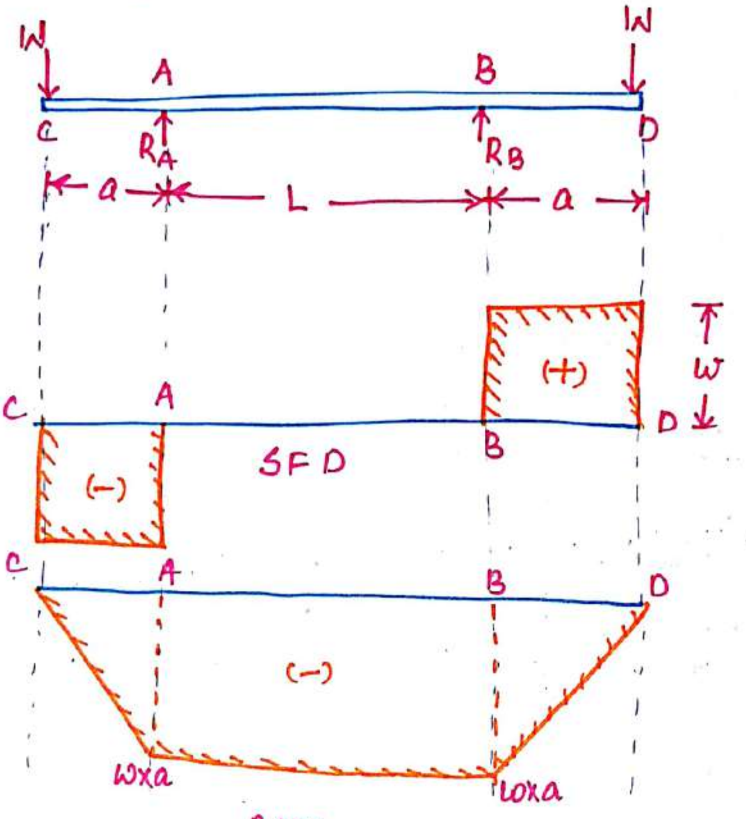
# UNIT - III

# FLEXURAL STRESSES

## 1. PURE BENDING OR SIMPLE BENDING :-

If a length of a beam is subjected to a constant bending moment and no shear force, then the stresses will set up in that length of the Beam due to B.M only.

The length of that beam is said to be in Pure bending or Simple bending.




BMD

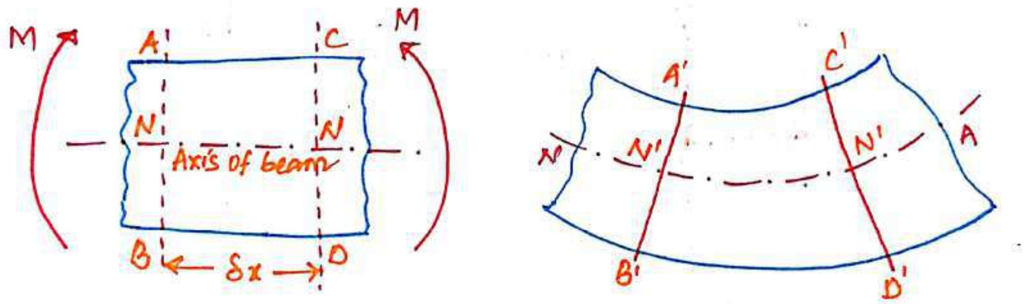
In the above SFD and BMD of the Beam, it is clear that the beam is subjected to a constant Bending moment b/w A & B.

This condition of the beam between A & B is known as pure bending or Simple Bending.

## 2. THEORY OF SIMPLE BENDING WITH ASSUMPTIONS MADE:

1. The material of the beam is homogenous & isotropic
2. The value of young's modulus of elasticity is same in both Tension and Compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The Beam is initially straight and all longitudinal filaments bend into circular arcs with common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross section.
6. Each layer of beam is free to expand or contract, independently of the layer, above or below it.

### Theory of Simple Bending:




* Consider a beam subjected to bending (simple bending).
* Consider a small length '$\delta x$' of this part of beam.
* Consider two sections AB & CD which are normal to the axis of the beam N–N.
* Due to the action of the bending moment, the part of length '$\delta x$' will be deformed.
* The top layer AC has deformed to a shape A'C' and bottom layer BD has deformed to B'D'

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* There will be a layer which is neither shortened nor elongated. This layer is known as "Neutral axis or neutral surface".

* The layers above Neutral axis have been shortened, these layers will be subjected to Compressive Stress. Due to increase in length of the below layers, these layers will be subjected to Tensile Stress.

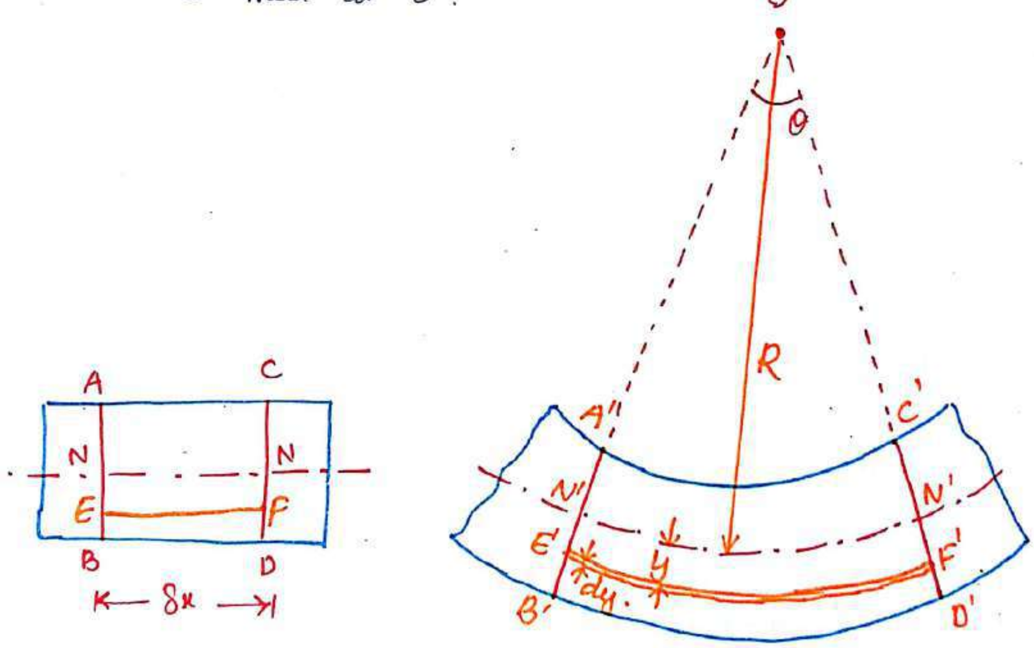
* The amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N-N.

* This theory of Bending is known as Theory of Simple Bending.

## 3. EXPRESSION FOR BENDING STRESS:-

Consider a small length '$\delta x$' of a beam subjected to a simple bending. Due to the action of bending, the part of length '$\delta x$' will be deformed as shown in figure.

Let A'B' and C'D' meet at 'O'.




### Strain Variation Along the Depth of Beam:-

Consider a layer EF at a distance 'y' below the neutral layer NN'. After bending this layer will be elongated to E'F'

Original length of layer, $EF = \delta x$

Also Length of neutral axis, $NN = \delta x$.

After Bending, the length of N.A ($N'N'$) will remain unchanged but the length of layer $E'F'$ will increase

$$\boxed{N'N' = NN = \delta x}$$

Now from figure

$$N'N' = R \times \theta \qquad [\because \text{Radius of } E'F' = R+y].$$

$$E'F' = (R+y)\theta$$

but $N'N' = NN = \delta x$

Hence $\delta x = (R+y)\theta$

$\therefore$ Increase in the length of the layer EF.

$$= E'F' - EF$$

$$= (R+y)\theta - R \times \theta$$

$$= y\theta.$$

$\therefore$ Strain in the layer EF $= \dfrac{\text{Increase in length}}{\text{Original length.}}$

$$= \frac{y \times \theta}{EF} = \frac{y \times \theta}{R \times \theta}$$

$$= \frac{y}{R}$$

As 'R' is Constant, hence the strain in a layer is proportional to its distance from the neutral axis.

The above equation shows the variation of strain along the depth of the beam.

The variation of strain is linear.

$$\boxed{\varepsilon \propto y.}$$

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## Stress Variation:

Let $\sigma$ = Stress variation in layer EF.

We know $E = \frac{\text{Stress in EF}}{\text{Strain in EF}} = \frac{\sigma}{(y/R)}$ $\left[\because \text{Strain in EF} = \frac{y}{R}\right]$.

$$\Rightarrow \sigma = E \times \frac{y}{R}$$

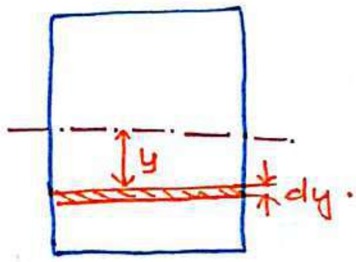
$$\boxed{\sigma = \frac{E}{R} \times y}$$

Since E & R are constant, the stress in any layer is directly proportional to the distance of the layer from the neutral axis.

In this case, all layers below the neutral axis are subjected to tensile stresses whereas the layers above are subjected to compressive stresses.

Consider the thickness of the small layer as 'dy'

let 'dA' = Area of the layer.




Now

The force of the layer

= Stress on layer × Area of layer.

$= \sigma \times dA$

$= \left(\frac{E}{R} \times y\right) \times dA$ $\left[\because \sigma = \frac{E}{R} \times y\right]$.

Total force on the beam section is obtained by integrating the above equation.

$\therefore$ Total force of the beam section

$$= \int \frac{E}{R} \times y \times dA$$

$$= \frac{E}{R} \int y \, dA$$

But for pure bending, there is no force on the section of the beam

$\therefore \frac{E}{R} \int y \times dA = 0$ $\left[\because \frac{E}{R} \neq 0\right]$.

$$\boxed{\int y \times dA = 0.}$$

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# Moment of Resistance:-

Due to pure bending, the layers above N.A. are Subjected to Compressive stresses whereas the layers below N.A. are Subjected to tensile stresses.

Due to these stresses, the forces will be acting on the layers. These forces will have moment about the N.A.

The total moment of these forces about the N.A. for a section is known as moment of resistance of that section.

$$\text{Force on that layer} = \frac{E}{R} \times y \times dA$$

Moment of this force about N.A.

$$= \text{Force on layer} \times y.$$

$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} \times y^2 \times dA$$

Total moment of the forces on the section of the beam (moment of resistance).

$$= \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R} \int y^2 dA.$$

For equilibrium the moment of resistance offered by the section should be equal to the external bending moment.

$$\boxed{M = \frac{E}{R} \int y^2 dA}$$

But, the expression $\int y^2 dA$ represents the moment of Inertia of the section about the neutral axis.

$$M = \frac{E}{R} \times I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}}$$

But from equation we have $\frac{\sigma}{y} = \frac{E}{R}$

$$\therefore \boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}} \Rightarrow * \boxed{\frac{E}{R} = \frac{M}{I} = \frac{f}{y}} *$$

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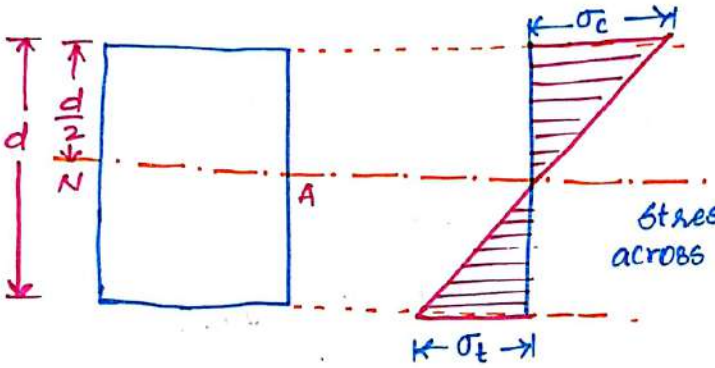
## 4. BENDING STRESSES IN SYMMETRICAL SECTIONS:

- The Neutral axis of a Symmetrical Section lies at a distance of $d/2$ from the outermost layer of the section where 'd' is diameter or depth.
- There is no stress at the Neutral axis. But the stress at a point is directly proportional to its distance from neutral axis.
- The maximum stress takes place at the outermost layer
- For a simply supported beam there is compressive stress above the neutral axis and a tensile stress below it.




Stress distribution across a section.

**Q:** A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2\times10^5 \, N/mm^2$.

**Sol:** Given

width of plate = 120 mm

thickness of plate = 20 mm

Radius of Arc = 10 m

$E = 2\times10^5 \, N/mm^2$

$\therefore$ Moment of Inertia, $I = \frac{bt^3}{12} = \frac{120\times20^3}{12} = 8\times10^4 \, mm^4$

Radius of Curvature, $R = 10m = 10\times10^3$ mm.

Young's mod Let

$\sigma_{max}$ = maximum instantaneous stress

$M$ = Bending moment.

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Using eq'n

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

$$\sigma = \frac{E}{R} \times y .$$

$$y_{max} = \frac{d}{2} = \frac{20}{2} = 10m .$$

$$\sigma_{max} = \frac{2 \times 10^5}{10 \times 10^3} \times 10 = \underline{200\ N/mm^2}\ \text{Ans.}$$

from eq'n

$$\frac{M}{I} = \frac{E}{R}$$

$$M = \frac{E}{R} \times I = \frac{2 \times 10^5}{10 \times 10^3} \times 8 \times 10^4$$

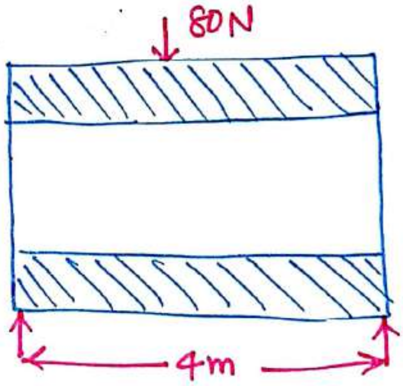
$$= 16 \times 10^5\ N\text{-}mm .$$

$$= \underline{1.6\ KNm .}\ \text{Ans.}$$

Q: Calculate the maximum stress induced in a cast iron pipe of external dia 40mm of internal dia 20mm and of length 4m when the pipe is supported at its ends and carries a point load of 80N at its centre.

Sol:- Given

External dia = 40mm
Internal dia = 20 mm
length. $L = 4m = 4 \times 10^3$ mm
point load, $W = 80$ KN



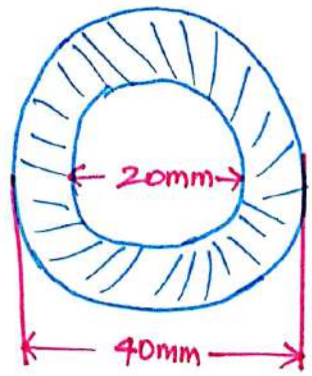

In case of simply supported beam, carrying a point load at the centre, the maximum B.M occur at centre of the beam

maximum

$$B.M = \frac{WL}{4}$$

$$= \frac{80 \times 4000}{4}$$

$$= 8 \times 10^4\ Nmm$$

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Moment of Inertia of the Hollow pipe

$$I = \frac{\pi}{4}(D^4 - d^4)$$

$$= \frac{\pi}{4}(40^4 - 20^4) = \frac{\pi}{4}(2560000 - 160000)$$

$$= 117809.7 \text{ mm}^4$$

Now using The equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\boxed{\sigma = \frac{M}{I} \times y}$$

$$y = y_{max}$$
$$= \frac{40}{2} = 20 mm$$

for maximum stress

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$= \frac{8 \times 10^4}{117809.7} \times 20$$

$$= \underline{13.58 \text{ N/mm}^2} \text{ Ans.}$$

Q: A rectangular beam 300 mm deep is Simply Supported Over a span of 4m. what amount of uniformly distributed load per metre, the beam may carry if the bending stress is not to exceed 120 N/mm$^2$. Take $I = 8 \times 10^6$ mm$^4$.

Ans:- 3.2 KN/m.

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## 5. SECTION MODULUS:

It is defined as the ratio of moment of inertia of a section about 'Neutral axis' to the outermost layer from the neutral axis.

It is donated by 'Z'

$$\boxed{Z = \frac{I}{y_{max}}}$$

Where $I$ - moment of inertia about Neutral axis.

$y_{max}$ - Distance of Outermost layer from N.A.

We know that

$$\frac{M}{I} = \frac{\sigma}{y}$$

The stress '$\sigma$' will be max if '$y$' is max

$$\Rightarrow \frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \cdot \frac{I}{y_{max}} \qquad \left[\because Z = \frac{I}{y_{max}}\right].$$

$$\boxed{M = \sigma_{max} \cdot Z.}$$

Hence,

moment of resistance offered the Section is maximum when Section modulus 'Z' is maximum.

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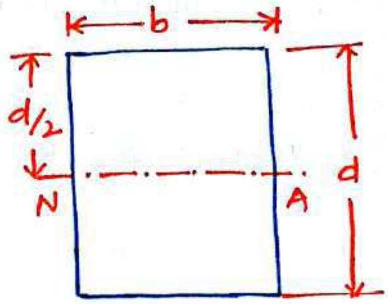
## 6. SECTION MODULUS FOR RECTANGULAR SECTION:




Moment of inertia of a rectangular Section about an axis through its C.G.

$$I = \frac{bd^3}{12}$$

Distance of outermost layer from N.A is given by

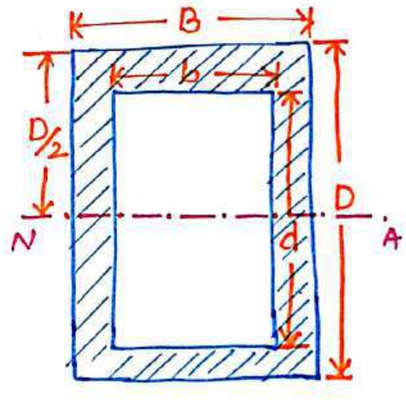
$$y_{max} = \frac{d}{2}$$

$\therefore$ Section modulus is given by

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times (\frac{d}{2})} = \frac{bd^3}{12} \times \frac{2}{d}$$

$$* \boxed{Z = \frac{bd^2}{6}} *$$

## 7. SECTION MODULUS FOR RECTANGULAR HOLLOW SECTION:




Moment of inertia

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}[BD^3 - bd^3].$$

Distance of outermost layer from N.A

$$y_{max} = \frac{D}{2}$$

$\therefore$ Section modulus is given by

$$Z = \frac{I}{y_{max}} = \frac{\frac{1}{12}[BD^3 - bd^3]}{\frac{D}{2}}$$

$$= \frac{1}{12}[BD^3 - bd^3] \times \frac{2}{D}$$

$$* \boxed{Z = \frac{[BD^3 - bd^3]}{6D}} *$$

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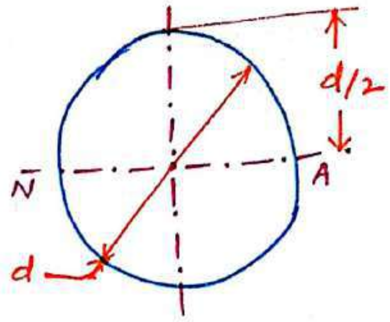
8. SECTION MODULUS FOR CIRCULAR SECTION:




moment of Inertia

$$I = \frac{\pi}{64} d^4$$

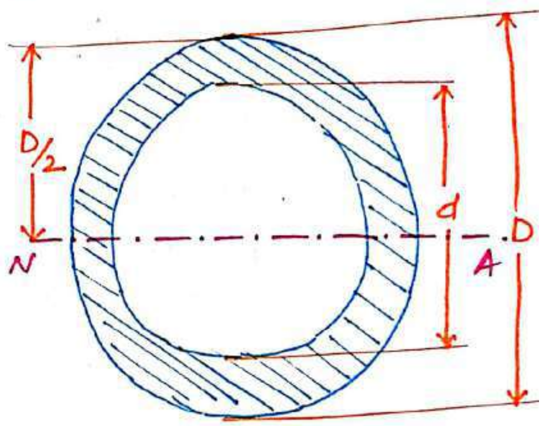
Distance of outermost layer from N.A

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \frac{\pi}{64} \times d^4 \times \frac{2}{d}$$

$$\boxed{Z = \frac{\pi}{32} d^3}$$

9. SECTION MODULUS FOR HALLOW CIRCULAR SECTION:




moment of inertia

$$I = \frac{\pi}{64} D^4 - \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (D^4 - d^4).$$

Distance of outermost layer from N.A.

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{D}{2}}$$

$$\boxed{Z = \frac{\pi}{32 D} [D^4 - d^4]}$$

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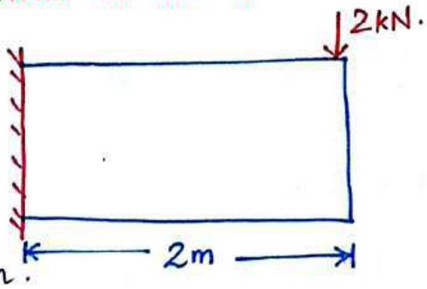
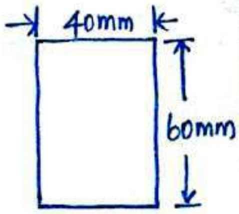
Q: A Cantilever of length 2m fails when a load of 2kN is applied at the free end. If the section of the beam is 40mm x 60mm, find the stress at the failure.

Sol: Given




Length, $L = 2m = 2\times10^3$ mm

Load, $W = 2$ kN $= 2\times10^3$ N

Section of beam is 40mm x 60mm.

∴ Width of beam, $b = 40$mm

depth of beam, $d = 60$ mm.

(a) Section modulus of a rectangular beam

$$Z = \frac{bd^2}{6} = \frac{40\times60^2}{6}$$

$$= 24000 \text{ mm}^3.$$

The stress at failure = maximum stress = $\sigma_{max}$.

maximum B.M for cantilever happens at fixed end

$$M = W\times L$$

$$= 2\times10^3\times2\times10^3 \text{ N-mm}$$

$$= 4\times10^6 \text{ N-mm}.$$

As we know.

$$\frac{M}{I} = \frac{\sigma}{y}$$

for max: $M = \sigma_{max}\cdot Z$

$$\sigma_{max} = \frac{M}{Z}$$

$$= \frac{4\times10^6}{24000} \frac{\text{N-mm}}{\text{mm}^3}$$

$$= 166.67 \text{ N/mm}^2.$$

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Q: A rectangular beam 200mm deep and 300mm wide is simply supported over a span of 8m. what uniformly distributed load per metre the beam may carry. if the bending stress is not to exceed 120 N/mm².

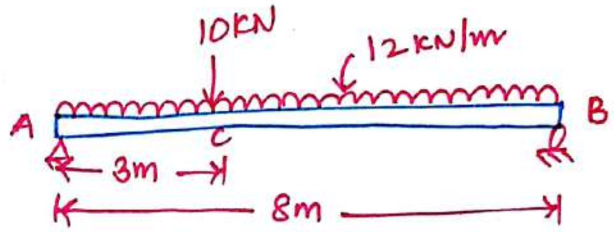
Q:. A timber beam of rectangular section of length 8m is simply supported. The beam carries a UDL of 12kN/m run over the entire span and a point load of 10KN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm², find the suitable dimensions of the section.

Sol: Given

length of beam, $l = 8m$

UDL $= 12 \ kN/m = 12 \times 10^3 \ N/m$

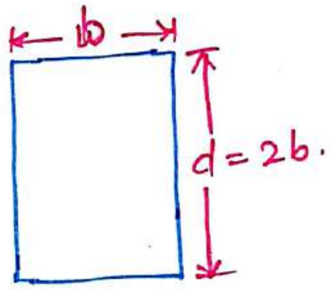
Point load $= 10 \ KN = 10 \times 10^3 N$.




depth of the beam, $d = 2 \times$ width of beam

$= 2 \times b$.

$\boxed{d = 2b}$




max stress, $\sigma_{max} = 8 N/mm^2$.

first calculate the section where B.M is maximum

Where B.M is maximum, the shear force will be zero. Now the equations of pure bending can be used.

For reactions at A & B, Now consider moments about A

$$R_B \times 8 = (12000 \times 8)\left(\frac{8}{2}\right) + 10000 \times 3$$

$$R_B = \frac{12000 \times 32 + 30000}{8} = 51750 \ N.$$

$$R_A = \text{Total load} - R_B = (12000 \times 8 - 10000) - 51750$$

$$= 54250N,$$

S.F.D:.

SF @ A = $+R_A$ = +54250 N.

SF @ LHS of C = 54250 − 12000×3

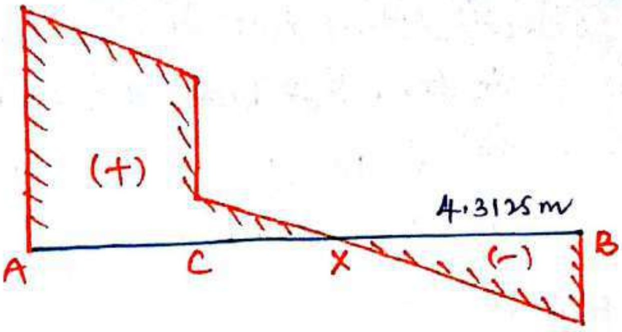
= +18250 N

SF @ RHS of C = 16250 − 1000

= +8250 N

SF @ B = 8250 − 12000×5

= −57750 N.




SF Changes its Sign between section CB, hence at some section b/w C&B the SF will be zero.

SF @ X-X, where SF=0 b/w C&B

$$12000 \times x - R_B = 0$$

$$x = \frac{57750}{12000} = 4.3125\,m.$$

∴ Maximum B.M will occur at 4.3125 from B.

∴ Maximum B.M, $M = R_B \times 4.3125 - 12000 \times 4.3125 \times \frac{4.3125}{2}$

$= 51750 \times 4.3125 - 111585.9375$

$= 111585.9375$ N-m

$= 111585.94 \times 10^3$ N-mm.

Section modulus for rectangular beam

$$Z = \frac{bd^2}{6} = \frac{b(2b)^2}{6} = \frac{2b^3}{3}$$

Now as we know

$$M = \sigma_{max} \cdot \frac{I}{y_{max}} \Rightarrow M = \sigma_{max} \cdot Z$$

$$11585.94 \times 10^3 = 8 \times \frac{2b^3}{3} \Rightarrow b^3 = \frac{3 \times 111585.94 \times 10^3}{16}$$

$$= 20.9223 \times 10^6$$

b = 275.5 mm

d = 2× 275.5 = 551 mm.

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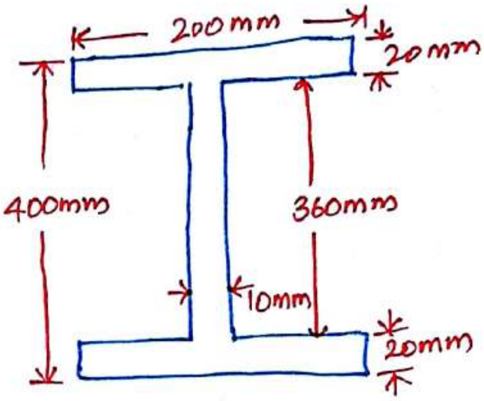
Q. A rolled steel joist of I section has the dimensions as shown in fig. This beam of I section carries a U.D.L of 40KN/m run on a span of 10m, calculate the maximum stress produced due to bending.




Sol: Given

U.D.L, $w = 40$ KN/m

$= 40\times10^3$ N/m

Span, $L = 10$m

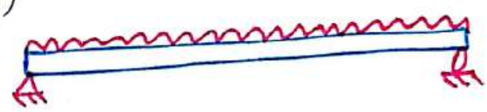
moment of inertia about N.A.

$$= \frac{200\times400^3}{12} - \frac{(200-10)\times360^3}{12}$$

$$= 3.28\times10^8 \text{ mm}^4$$

max. B.M, $M = \frac{wL^2}{8}$ [∵ The beam section is given as S.S. beam with U.D.L]

$$= \frac{40\times10^2}{8} = \frac{40\times10^3\times(10\times10^3)^2}{8}$$



$$= 5\times10^8 \text{ Nmm}$$

$y_{max} = 200$ mm.

Now using the relation

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I}\times y$$

$$\sigma_{max} = \frac{M}{I}\times y_{max}$$

$$= \frac{5\times10^8}{3.28\times10^8}\times200$$

$$= 304.92 \text{ N/mm}^2.$$

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Q: A water main of 500mm internal dia and 20mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weighs 72000N/m³ & 10000 N/m³ respectively.

Sol:- Given

Internal dia = 500mm

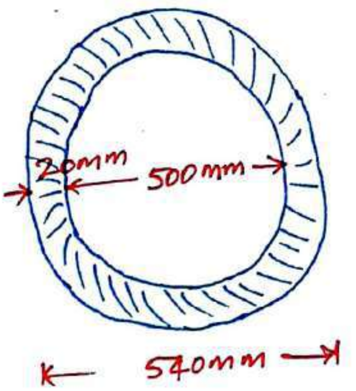
thickness = 20mm

∴ Ext. dia = 500+ 2×20 = 540mm

Span = 10m = $10 \times 10^3$ mm

Weight density of cast iron = 72000 N/m³.

Weight density of water = 10000 N/m³.




Moment of inertia of pipe section about N.A.

$$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right] = \frac{\pi}{64}\left[540^4 - 500^4\right]$$

$$= 1.105 \times 10^9 \text{ mm}^4$$

Now, let us find weight of pipe & water per unit metre.

Weight of pipe per m' length = wt. density of iron × volume

$$= 72000 \times [\text{Area of pipe} \times \text{length}]$$

$$= 72000 \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right] \times 1$$

$$= 2354 \text{N}.$$

Weight of water per m length = $10000 \times \frac{\pi}{4} \times D_i^2 \times 1$

$$= 1960 \text{N}.$$

∴ Total wt of pipe for 1m run = 2354 + 1960 = 4314 N.

$$\therefore \text{Max. BM} = \frac{wL^2}{8} = \frac{4314 \times 10^2}{8} = 53925 \times 10^3 \text{Nmm}$$

$$\because \frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I} \times y$$

$$= \frac{53925 \times 10^3}{1.105 \times 10^9} \times 270 \qquad (\because y_{max} = 270 \text{ mm})$$

$$= 13.18 \text{ N/mm}^2.$$

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## 10. BENDING STRESSES IN UNSYMMETRICAL SECTIONS:

In case of unsymmetrical sections like L, T, the neutral axis doesn't pass through the geometrical centre of the section

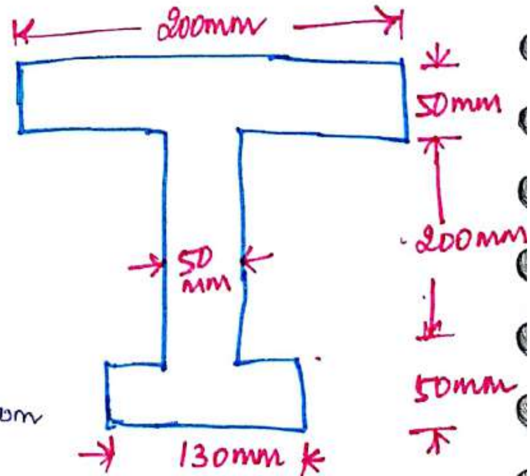
Hence the value

Q: A cast iron bracket subjected to bending has the c/s of I-form with unequal flanges. The dimensions of the section are shown in the figure. find the position of the neutral axis and moment of inertia of the section about N.A. If the maximum B.M on the section is 40MN mm. determine the maximum bending stress. what is the nature of the stress?

Sol: Given

BM, $M = 40$ MN mm

$= 40 \times 10^6$ Nmm




Let us calculate the C.G of the section.

Let $\bar{y}$ is the distance of C.G from the bottom face.

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$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$

$$= \frac{\left[(130\times 50)\times \frac{50}{2}\right] + \left[(200\times 50)\times\left(50+\frac{200}{2}\right)\right] + \left[(200\times 50)\times\left(50+200+\frac{50}{2}\right)\right]}{(130\times 50)+(200\times 50)+(200\times 50)}$$

$$= \frac{4412500}{26500} = 166.51\,mm.$$

Hence N.A is @ 166.51 mm from the bottom face.

Moment of Inertia of the section about N.A.

$$I = I_1 + I_2 + I_3.$$

$I_1 =$ M.O.I of bottom flange about axis passing through its C.G $+ A_1 \times y_1^2$

$$= \frac{130\times 50^3}{12} + (130\times 50)(166.51-25)^2$$

$$= 131517186.6\ mm^4$$

$$I_2 = \frac{50\times 200^3}{12} + (50\times 200)\times(166.51-150)^2 = 33605913.43\ mm^4$$

$$I_3 = \frac{200\times 50^3}{12} + (200\times 50)\times(275-166.51)^2 = 119784134.3\ mm^4.$$

$$I = I_1 + I_2 + I_3 = 284907234.9\ mm^4,$$

Now

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I}\times y = \frac{40\times 10^6}{284907234.9}\times 166.51 = 23.377\ N/mm^2$$

$\therefore$ Maximum Bending Stress

$$= 23.377\ N/mm^2.$$

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# 11. SHEAR STRESSES IN BEAMS:-

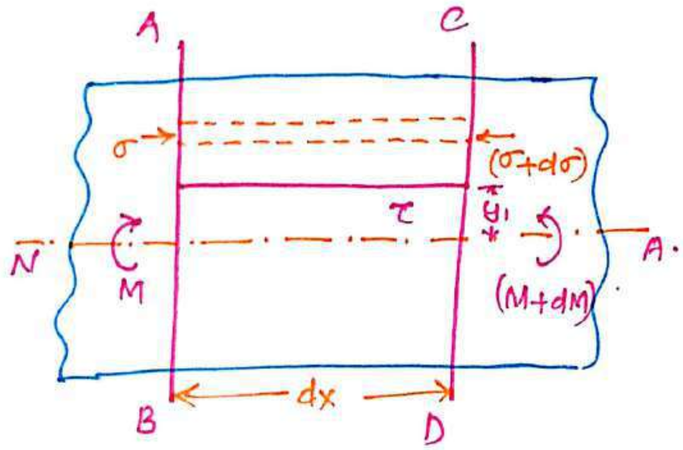
When a part of beam is subjected to a constant bending moment and zero shear force, then there will be only bending stresses in the beam.

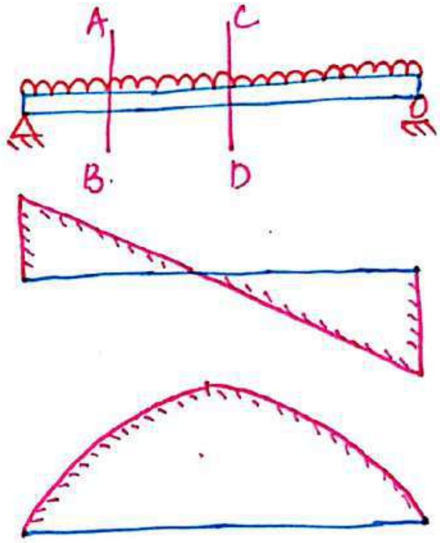
In actual practice, a beam is subjected to a B.M varies from section to section. Also the S.F acting on the beam is not zero.

Due to the shear forces, the beam will be subjected to shear stresses.

## SHEAR STRESS AT A SECTION:-

Consider a simply supported beam carrying a U.D.L.




Consider two sections AB & CD of this beam at a distance 'dx' apart

Let at section 'AB', Shear force $= F$

Bending moment $= M$.

and at section 'CD', Shear force $= F + dF$

Bending moment $= M + dM$

$I$ = moment of inertia of the section about N.A.

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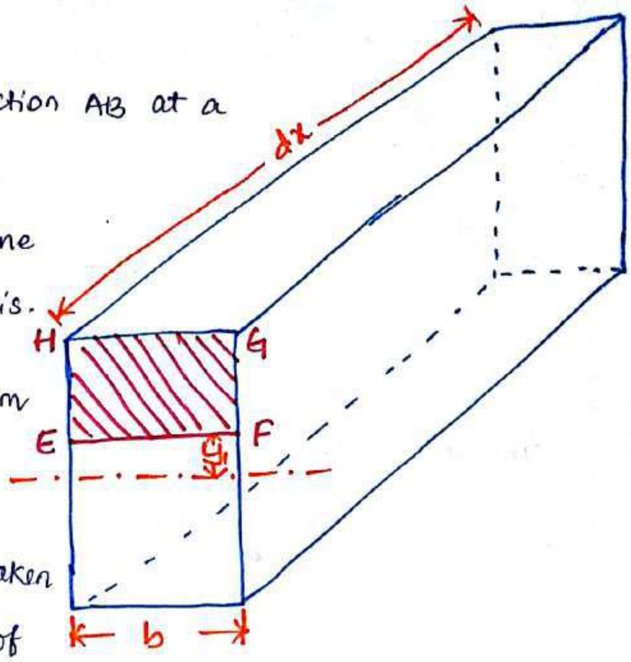
It is required to find the shear stress on the section 'AB' at a distance '$y_1$' from the N.A.

To find the shear stress on the section AB at a distance $y_1$ from the Neutral axis.

On the cross-section EF be the line at a distance '$y_1$' from the Neutral axis.




Now consider the part of the beam above the level EF and b/w the sections AB & CD.

This part of the beam may be taken to consists of an infinite number of elemental cylinders of area 'dA' and length 'dx'.

Consider one such elemental cylinder

dA - Area of elemental cylinder

dx - length of the elemental cylinder

y - distance of the elemental cylinder from N.A

Let '$\sigma$' = intensity of bending stress on section AB

'$\sigma + d\sigma$' = intensity of bending stress on section CD.

The bending stress at a distance 'y' from N.A is given by

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I} \times y.$$

Similarly bending stress on the end of the cylinder on the section

$$\sigma + d\sigma = \frac{(M+dM)}{I} \times y.$$

Now let us find the forces on the two ends of the elemental cylinder

Force on the end of the elemental cylinder on section AB

= Stress X Area of elemental cylinder

$= \sigma \times dA.$

$= \frac{M}{I} \times y \times dA$ $\quad \left[ \because \sigma = \frac{M}{I} \times y \right]$

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Similarly, force on the elemental cylinder on the section CD.

$$= (\sigma + d\sigma)\, dA$$

$$= \frac{M + dM}{I} \times y \times dA.$$

∴ Net unbalanced force on the elemental cylinder

$$= \frac{(M+dM)}{I} \times y \times dA - \frac{M}{I} \times y \times dA$$

$$= \frac{dM}{I} \times y \times dA.$$

The total unbalanced force above the level EF

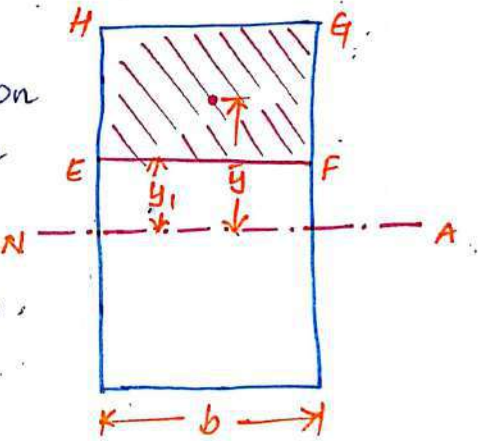
$$= \int \frac{dM}{I} \times y \times dA$$

$$= \frac{dM}{I} \int y\, dA$$

$$= \frac{dM}{I} \times A \times \bar{y}$$

A - Area of the EFGH as shown

$\bar{y}$ - Distance of the C.G of the Area 'A' from N.A.




Due to the total unbalanced force on the beam above the level EF, the beam may fail due to shear

In order to prevent from shear failure, the horizontal section of the beam must offer a shear resistance.

The shear resistance atleast must be equal to total unbalanced force to avoid failure due to shear.

∴ Shear resistance at level EF (or) Shear force = Total unbalanced force

$$= \frac{dM}{I} \times A \times \bar{y}$$

Let

$\tau$ = Intensity of horizontal Shear at level EF

$b$ = width of the beam at the level EF.

$\therefore$ Area on which '$\tau$' is acting = $b \times dx$.

$\therefore$ Shear force due to '$\tau$' = Shear Stress x Shear Area.

$= \tau \times b \times dx$

Now equating Shear forces

$$\tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

$$\tau = \frac{dM}{I} \times \frac{A \times \bar{y}}{b \times dx}$$

$$\tau = \frac{dM}{dx} \cdot \frac{A \times \bar{y}}{I \times b} \qquad \left[\because \frac{dM}{dx} = \text{Shear force}\right].$$

$$\boxed{\tau = F \frac{A\bar{y}}{I \times b}}$$

The shear stress given by the equation is the horizontal shear stress at the distance '$y_1$' from the Neutral axis.

Some times $A \times \bar{y}$ is also expressed as the moment of Area 'A' about the N.A.

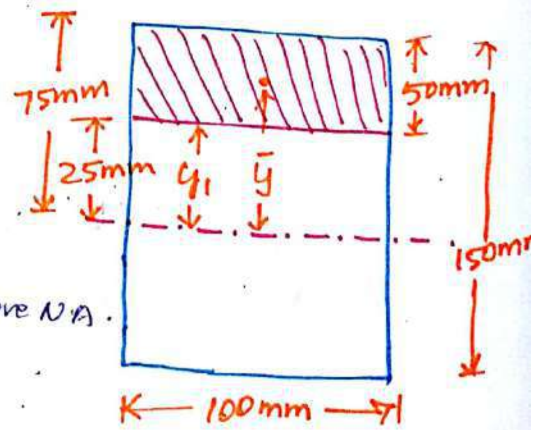
Q: A wooden beam 100 mm wide and 150 mm deep is simply supported over a span of 4m: If S.F at a section of the beam is 4500N. Find the Shear force at a distance of 25mm above the N.A.

Sol: Given

Width, $b = 100$ mm

Depth, $d = 150$ mm

Shear force, $F = 4500$ N




Let $\tau$ = Shear ~~force~~ stress at a distance 25mm above N.A.

Now

$$\tau = F \cdot \frac{A\bar{y}}{I \times b}$$

$$= 4500 \frac{(100 \times 150) \times (25 + \frac{50}{2})}{\frac{100 \times 150^3}{12} \times 100} = 0.4 \text{ N/mm}^2.$$

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## 12. SHEAR STRESS DISTRIBUTION FOR DIFFERENT SECTIONS:

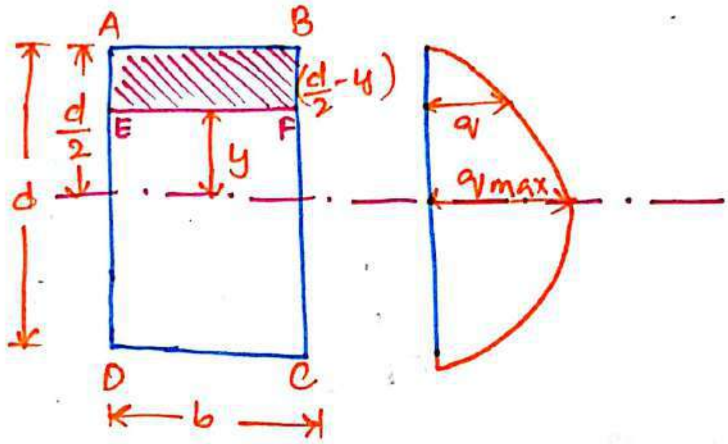
### i, RECTANGULAR SECTION:-

Consider a rectangular section beam of width 'b' and depth 'd'.

Let 'F' is the shear force acting at the section. Consider a level 'EF' at a distance 'y' from N.A.

The shear stress at this level

$$\boxed{\tau = F \cdot \frac{A\bar{y}}{b \times I}}$$




where

A = Area of the section above 'y'

$$\boxed{A = \left(\frac{d}{2} - y\right) \times b}$$

$\bar{y}$ = Distance of the C.G of Area 'A' from N.A.

$$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = \boxed{\frac{1}{2}\left[y + \frac{d}{2}\right]}$$

b = Actual width of the section at level EF = b.

I = M.O.I of whole section about N.A.

Substituting these values

$$\tau = F \cdot \frac{\left[\frac{d}{2} - y\right] \times b \times \frac{1}{2}\left[y + \frac{d}{2}\right]}{b \times I}$$

$$\boxed{\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]}$$

The variation of 'τ' with respect to 'y' is a parabola.

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Shear stress at the top edge, $y = \frac{d}{2}$

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - \frac{d^2}{4}\right] = 0$$

at Neutral axis, $y = 0$

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - 0\right] = \frac{F}{2I} \times \frac{d^2}{4}$$

$$= \frac{Fd^2}{8I} = \frac{Fd^2}{8 \times \frac{bd^3}{12}}$$

$$\boxed{\tau = 1.5\frac{F}{bd}.}$$

Now
Average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}}$

$$= \frac{F}{b \times d}$$

Substituting the above equation.

$$\boxed{\tau = 1.5\,\tau_{avg}.}$$

Q. A rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50 kN. Determine

a) Average shear stress

b) Maximum shear stress.

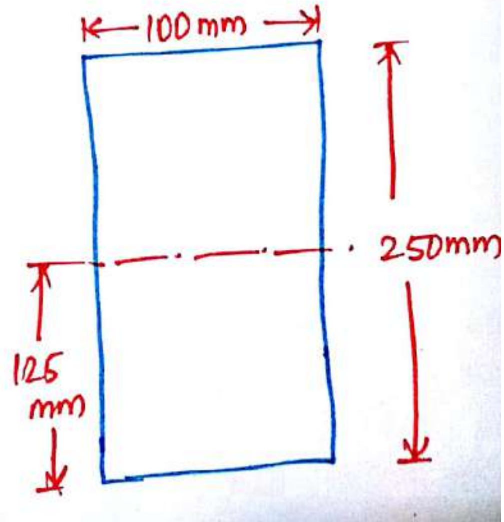
c) Shear stress at a distance of 25m above N.A.

Sol: Given

Width, $b = 100mm$

Depth, $d = 250mm$

max. shear force, $F = 50kN = 50{,}000N$.

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i, Average shear stress is given by

$$\tau_{avg} = \frac{F}{Area} = \frac{50,000}{b \times d}$$

$$= \frac{50,000}{100 \times 250} = 2\ N/mm^2.$$

ii, Maximum shear stress is given by.

$$\tau_{max} = 1.5 \times \tau_{avg}$$

$$= 1.5 \times 2 = 3\ N/mm^2.$$

iii, The shear stress at a distance 'y' from N.A. ($y = 25mm$)

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]$$

$$= \frac{50000}{2I}\left[\frac{250^2}{4} - 25^2\right]$$

$$= \frac{50000}{2 \times \frac{bd^3}{12}}\left[\frac{62500}{4} - 625\right]$$

$$= \frac{50000 \times 12}{2 \times 100 \times 250^3} \times 15000$$

$$= 2.88\ N/mm^2.$$

$$\tau = F \times \frac{A \times \bar{y}}{I \times b}$$

$$= 50,000 \times \frac{(100 \times 100) \times (75)}{\left[\frac{100 \times 250^3}{12}\right] \times 100}$$

$$= \frac{50000 \times 10000 \times 75 \times 12}{100 \times 250^3 \times 100}$$

$$= 2.88\ N/mm^2.$$

Q: A Simply Supported wooden beam of Span 1.3m having a C/s 150mm x 250mm deep Carries a point load 'W' at the centre. The permissable stress are $7\ N/mm^2$ in bending and $1\ N/mm^2$ in Shearing. Calculate the Safe load 'W'.

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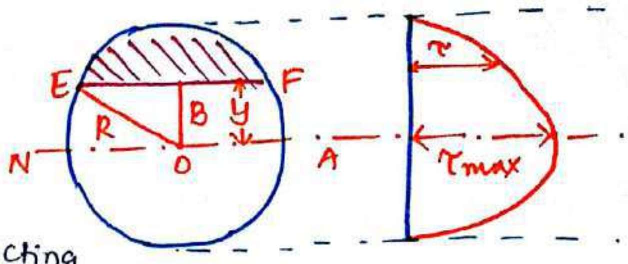
## ii) CIRCULAR SECTION:-

Consider a ~~Hallow~~ Circular Section of radius 'R'.




Let 'F' is the shear force acting on the section.

Take a level EF at a section 'y' from the Neutral axis.

Shear Stress at this level

$$\tau = F\frac{A\bar{y}}{Ib}$$

Consider a strip of thickness 'dy' at a distance 'y' from N.A.

let 'dA' is the area of strip.

$$dA = b \times dy$$

$$= EF \times dy = 2 \times EB \times dy$$

$$= 2 \times \sqrt{R^2-y^2} \times dy.$$

Moment of this area 'dA' about NA.

$$= y \times dA = y \times 2 \times \sqrt{R^2-y^2} \times dy.$$

$$= 2y\sqrt{R^2-y^2}\, dy.$$

moment of the whole section shaded above N.A.

$$A\bar{y} = \int_y^R 2y\sqrt{R^2-y^2}\, dy.$$

$$= -\int_y^R (-2y)\sqrt{R^2-y^2}\, dy.$$

Now $(-2y)$ is the differential of $(R^2-y^2)$, Hence integration of the above equation becomes.

$$A\bar{y} = \left[\frac{(R^2-y^2)^{3/2}}{3/2}\right]_y^R$$

$$= -\frac{2}{3}\left[(R^2-R^2)^{3/2} - (R^2-y^2)^{3/2}\right]$$

$$= -\frac{2}{3}\left[0 - (R^2-y^2)^{3/2}\right]$$

$$= \frac{2}{3}(R^2-y^2)^{3/2}.$$

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Substituting the values of $A\bar{y}$

$$\tau = \frac{F \times \frac{2}{3}(R^2-y^2)^{3/2}}{I \times b}$$

$$b = EF = 2 \times EB = 2 \times \sqrt{R^2-y^2}$$

$$\therefore \quad \tau = \frac{\frac{2}{3} F (R^2-y^2)^{3/2}}{I \times 2\sqrt{R^2-y^2}}$$

$$\boxed{\tau = \frac{F}{3I}(R^2-y^2)}$$

The equation shows that shear distribution across a circular section is parabolic.

at $y=0$ : at the N.A; the shear stress is maximum

$$\tau_{max} = \frac{F}{3I}(R^2-0^2)$$

$$\boxed{\tau_{max} = \frac{F}{3I}(R^2)} \Rightarrow \frac{F}{3 \times \frac{\pi}{64} D^4} \Rightarrow \boxed{\tau_{max} = \frac{4F}{3\pi R^2}}$$

Average shear stress.

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area of circular section}}$$

$$= \frac{F}{\frac{\pi}{4} D^2}$$

$$= \frac{F}{\pi R^2}.$$

Hence

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{avg}.}$$

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Q: A Circular beam of 100 mm diameter is subjected to a shear force of 5KN. Calculate

i) Average Shear Stress

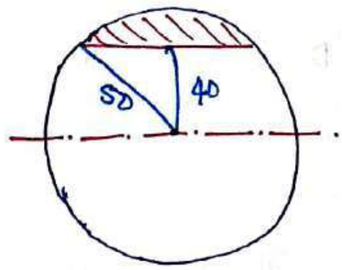
ii) Maximum Shear Stress.

iii) Shear Stress at a distance of 40 mm from N.A.

Sol: Given

Dia, $D = 100\,mm$

$\therefore$ Radius, $R = 50\,mm$.




Shear force, $F = 5KN$

$= 5000N$.

i) Average Shear Stress, $\tau_{avg} = \dfrac{\text{Shear force}}{\text{Area}}$

$$= \frac{5000}{\frac{\pi}{4} \times 100^2} = 0.6366\ N/mm^2.$$

ii) Maximum Shear Stress for a Circular Section

$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

$$= \frac{4}{3} \times 0.6366$$

$$= 0.8488\ N/mm^2.$$

iii) The Shear stress at a distance of 40mm from N.A.

$$\tau = \frac{F}{3I}(R^2 - y^2).$$

$$= \frac{5000}{3 \times \frac{\pi}{64} \times 100^2}\left[50^2 - 40^2\right]$$

$$= \frac{5000 \times 64}{3 \times \pi \times 100^2}\left[2500 - 1600\right]$$

$$= 0.3055\ N/mm^2.$$

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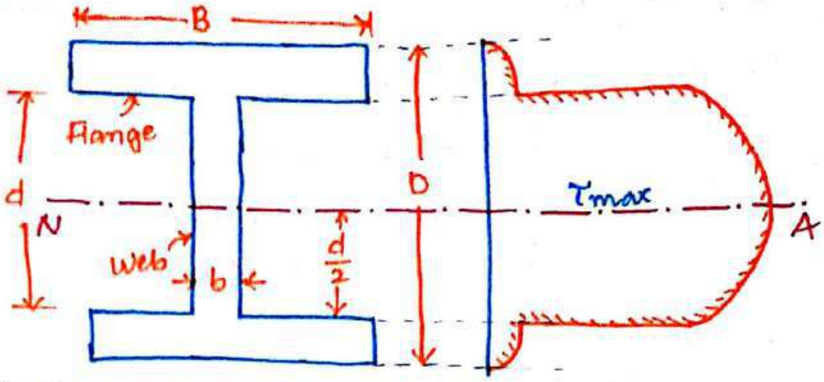
iii, I - SECTION :-




Consider a I-Section with

B - Overall width of the section.

D - Overall depth of the section

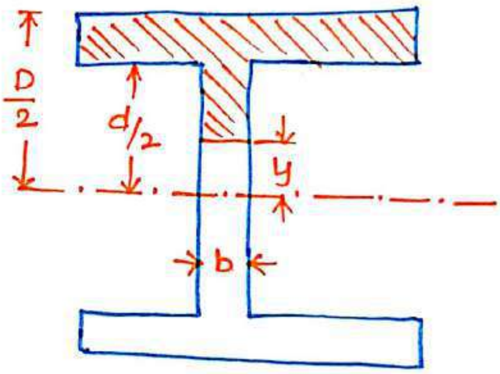
b - Thickness of the web, &

d - Depth of the web.

The Shear Stress at a distance 'y' from N.A.

$$\tau = F \times \frac{A\bar{y}}{I \times b}.$$

i, Shear Stress distribution in the flange.




Shaded area of the flange, $A = B\left[\frac{D}{2} - \frac{d}{2}\right]$.

Distance of the C.G of the Shaded flange from N.A

$$\bar{y} = y + \frac{1}{2}\left[\frac{D}{2} - \frac{d}{2}\right]$$

$$= \frac{1}{2}\left[\frac{D}{2} + \frac{d}{2}\right].$$

Hence shear stress in the flange

$$\tau = F \cdot \frac{A\bar{y}}{I \times b} = F \cdot \frac{B\left[\frac{D}{2} - \frac{d}{2}\right]\frac{1}{2}\left[\frac{D}{2} + \frac{d}{2}\right]}{I \times B}$$

$$= \frac{F}{2I}\left[\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$= \frac{F}{8I}\left[D^2 - d^2\right].$$

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ii, Shear distribution in the web :

Here moment area of the flange and web taken into consideration

$$\therefore A\bar{y} = \text{Moment of the flange about N.A} + \text{Moment of the shaded area of web about N.A.}$$

$$= B\left[\frac{D}{2}-\frac{d}{2}\right]\times\frac{1}{2}\left[\frac{D}{2}+\frac{d}{2}\right] + b\left[\frac{d}{2}-y\right]\times\frac{1}{2}\left[\frac{d}{2}+y\right]$$

$$= \frac{B}{8}(D^2-d^2) + \frac{b}{2}\left[\frac{d^2}{4}-y^2\right].$$

$\therefore$ Shear Stress in the web becomes

$$\boxed{\tau = \frac{F\times A\bar{y}}{I\times b} = \frac{F}{I\times b}\times\left[\frac{B}{8}(D^2-d^2)+\frac{b}{2}\left(\frac{d^2}{4}-y^2\right)\right]}$$

At the neutral axis, $y=0$ hence shear stress is maximum

$$\tau_{max} = \frac{F}{I\times b}\left[\frac{B}{8}(D^2-d^2)+\frac{b}{2}\left(\frac{d^2}{4}-0\right)\right]$$

$$\boxed{\tau_{max} = \frac{F}{I\times b}\quad\frac{B(D^2-d^2)}{8}+\frac{bd^2}{8}}$$

At the junction of top of the web and bottom of flange $y=\frac{d}{2}$

$$\tau = \frac{F}{I\times b}\left[\frac{B}{8}(D^2-d^2)+\frac{b}{2}\left[\frac{d^2}{4}-\frac{d^2}{4}\right]\right].$$

$$= \frac{F\times B\times(D^2-d^2)}{8I\times b.}$$

The shear stress at the junction of the flange and the web changes abruptly.

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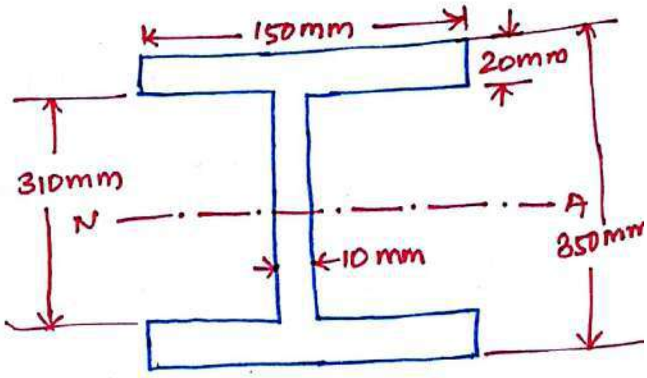
**Q:** An I Section beam 350 mm × 150 mm has a web thickness of 10mm and a flange thickness of 20mm. If the shear force acting on the section is 40 kN. Find the maximum shear stress developed in the section.




**Sol:** Overall depth, $D = 350$ mm

Overall width, $B = 150$ mm

Web Thickness, $b = 10$ mm

Flange Thickness $= 20$ mm.

$\therefore$ Depth of web, $d = 350 - (2 \times 20)$

$= 310$ mm.

Shear force on the Section, $F = 40$ kN $= 40{,}000$ N.

Moment of Inertia of the section about N/A.

$$I = \frac{150 \times 350^3}{12} - \frac{140 \times 310^3}{12} = 188375833.4 \text{ mm}^4$$

Moment area $A \times \bar{y}$ = Area of the flange × Distance of C.G of the area of flange + Area of web above NA × distance of the C.G of the area from N.A.

$$= (150 \times 20) \times \left(\frac{310}{2} + \frac{20}{2}\right) + \left(\frac{310}{2} \times 10\right) \times \left(\frac{310}{2} \times \frac{1}{2}\right)$$

$$= 615125 \text{ mm}^3.$$

$$\tau_{max} = \frac{40{,}000 \times 615125}{188375833.4 \times 10}$$

$$= 13.06 \text{ N/mm}^2.$$

**Shear Stress distribution in the flange:**

The shear stress at the upper edge of the flange is zero.

$\because y = \frac{D}{2}$ $\quad \tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right] = 0.$

for the lower edge of the upper flange, at the joint of the web and flange

$y = \frac{d}{2}.$

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$$\therefore \tau = \frac{F}{2I}\left[\frac{D^2}{4}-\left(\frac{d}{2}\right)^2\right] = \frac{F}{2I}\left[\frac{D^2}{4}-\frac{d^2}{4}\right]$$

$$= \frac{F}{8I}(D^2-d^2) = \frac{40000}{8\times 188.375\times 10^6}(350^2-310^2) = 0.7007\ N/mm^2.$$

**Shear Stress distribution in the web:-**

The shear stress is maximum at N.A and it is given by.

$$\tau_{max} = 13.06\ N/mm^2.$$

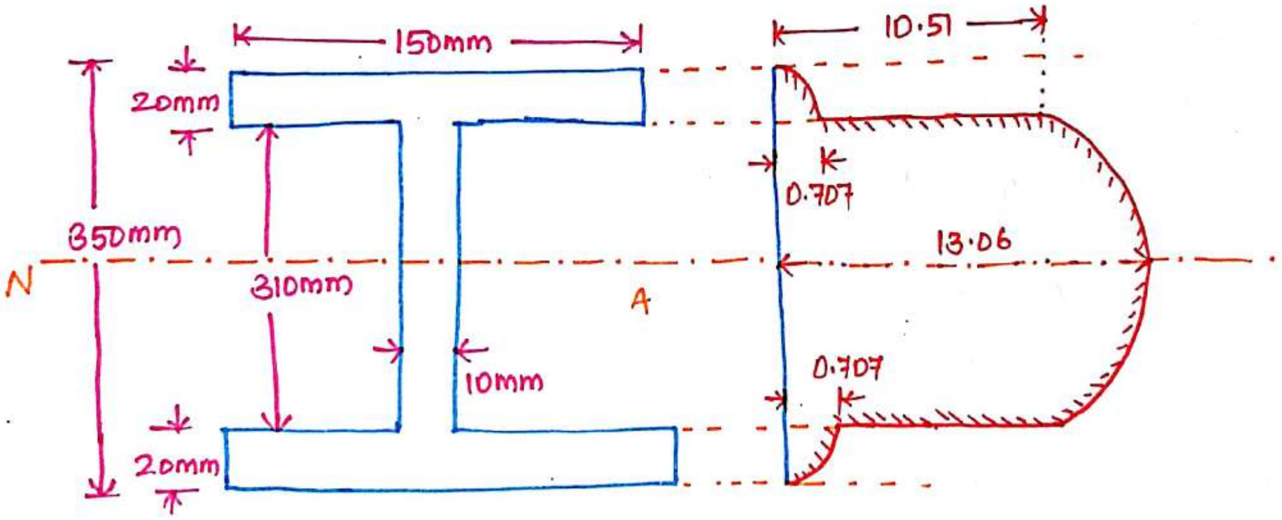
The shear stress at the junction of web and flange is given by the equation

$$\tau = \frac{F\times B}{8I\times b}(D^2-d^2)$$

$$= \frac{40{,}000}{8\times 188.375\times 10}(350^2-310^2)$$

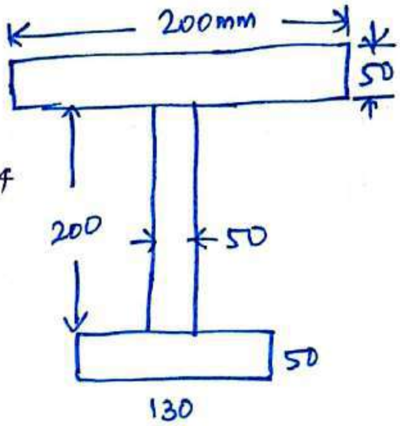
$$= 10.51\ N/mm^2.$$

Now the shear stress distribution which is symmetrical about N.A The shear stress for web and flange are parabolic. The shear stress at the junction suddenly changes from 0.707 to $10.51\ N/mm^2$



Q: The shear stress acting on a beam at an I-section with unequal flanges is 50 KN. The section is shown in fig. The moment of inertia of the section about N.A is $2.849 \times 10^4$. Calculate the shear stress at the N.A and also draw the shear stress distribution over the depth of the section.

Sol: Given




Shear force, $F = 50KN = 50,000 N$.

moment of inertia about N.A $\Rightarrow I = 2.849 \times 10^8 mm^4$

Let us calculate the position of N.A.

$$y^* = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{\left[(130 \times 50) \times \left(\frac{50}{2}\right)\right] + \left[(200 \times 50) \times \left(50 + \frac{200}{2}\right)\right] + \left[(200 \times 50) \times \left(50 + 200 + \frac{50}{2}\right)\right]}{(130 \times 50) + (200 \times 50) + (200 \times 50)}$$

$= 166.51 mm$

Hence N.A is at distance of 166.51mm from bottom & 133.49 from top.

Shear stress distribution:

i, Shear stress at the extreme edges of the flanges $= 0$.

ii, Shear stress in the upper flange just at the junction of upper flange and web is given by.

$$\tau = F \cdot \frac{A\bar{y}}{Ib}$$

$A\bar{y}$ = Area of upper flange x C.G of upper flange from N.A.

$= (200 \times 50) \times (133.49 - 25) = 1084900$.

$$\tau = 50000 \cdot \frac{1084900}{2.849 \times 10^8 \times 200}$$

$= 0.9520 N/mm^2$.

iii, The shear stress in the web just at the junction of the web and upper flange will suddenly increase to

$$0.9520 \times \frac{200}{50}$$

$= 3.808 N/mm^2$,

$\left[\because \tau = F \cdot \frac{A\bar{y}}{Ib}\right]$

$b \Rightarrow 200$ for flange

$= 50$ for web

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iv, Shear Stress will be max at N·A, which is given by.

$$\tau_{max} = F \cdot \frac{A\bar{y}}{Ib}$$

$A\bar{y}$ = $A\bar{y}$ of upper flange + $A\bar{y}$ of web about NA

$$= (200\times50)\times(133{\cdot}49-25)+(133{\cdot}49-50)\times50\times\frac{(133{\cdot}49-50)}{2}$$

$$= 1084900 + 174264{\cdot}5 = 1259164{\cdot}5$$

$$\tau_{max} = 50{,}000 \times \frac{1259164{\cdot}5}{2{\cdot}849\times10^8\times50} = 4{\cdot}4196\ N/mm^2.$$

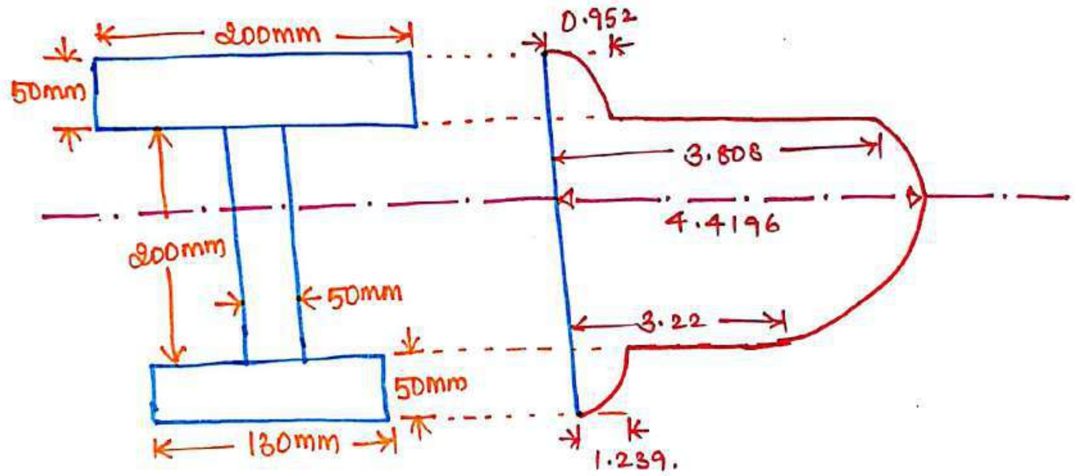
v, Shear stress at lower flange just at the junction of the lower flange and web

$$\tau = 50{,}000 \times \frac{(130\times50)\times(166{\cdot}51-25)}{2{\cdot}849\times10^8\times130}$$

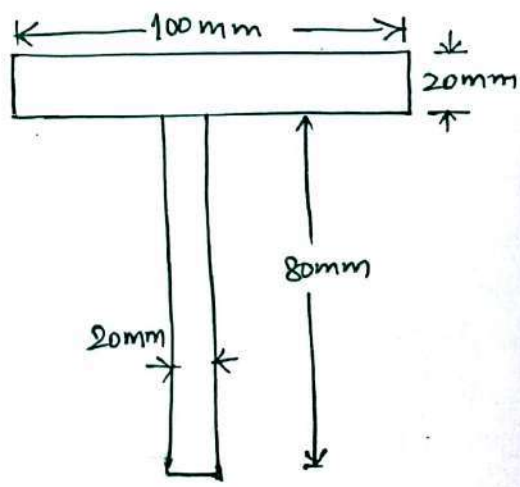
$$= 1{\cdot}239\ N/mm^2.$$

vi, The shear stress in the web just at the junction of the web and the lower flange increase from 1·239 to

$$1{\cdot}239\times\frac{130}{50} = 3{\cdot}22\ N/mm^2.$$




Q: The shear force acting on a section of beam is 50 KN. The section is T shaped and the moment of Inertia about Horizontal N·A is $314.221\times10^4\ mm^4$. Calculate the shear stress at the neutral axis and at the junction of the web and the flange.

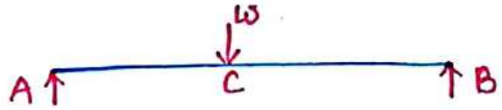


# UNIT - IV

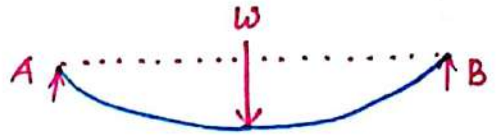
# DEFLECTION OF BEAMS

* If a beam carries a uniformly distributed load or a point load, the beam is deflected from its original position.

* Due to loads acting, it will subject to Bending moment




* The radius of curvature of the deflected beam is $\frac{M}{I} = \frac{E}{R}$




Where

$$R = \frac{(I \times E)}{M}$$

* The term $(I \times E)/M$ is constant, if the beam is subjected to a constant bending moment 'M'.

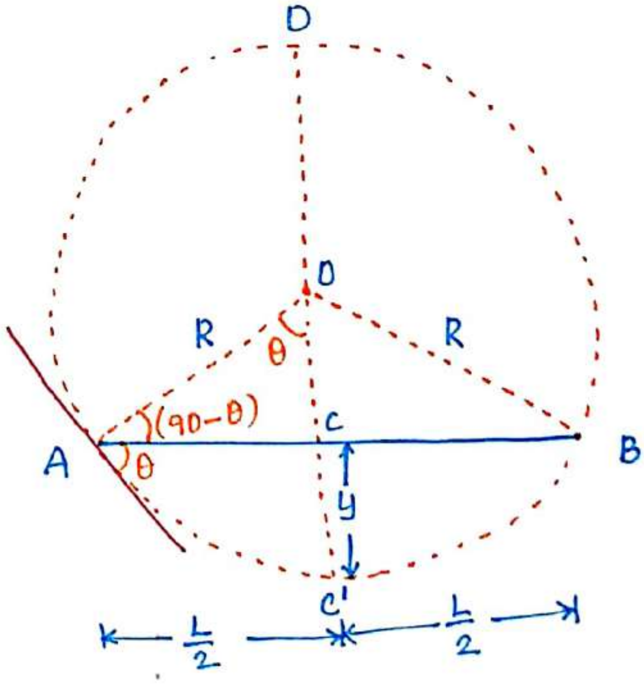
* This means that a beam for which, when loaded, the value of $(E \times I)/M$ is constant, will bend in a circular arc.

## 1. SLOPE & DEFLECTION OF A BEAM SUBJECTED TO UNIFORM BENDING MOMENT:

A beam AB of length 'L' is subjected to a Uniform Bending moment 'M'

As the beam is subjected to a constant B.M, hence it will bend into a circular arc.

The initial position of the beam is shown by ACB, whereas the deflected position is shown by AC'B.



Let

R - Radius of Curvature of the deflected beam.

y - Deflection of beam at the Centre.

I - Moment of inertia of the beam Section.

E - Youngs modulus for the beam material.

$\theta$ - Slope of the beam at the end 'A'.

for a practical beam the deflection 'y' is a small quantity.

Hence

$\tan\theta = \theta$ , where $\theta$ is in radians.

Hence

'$\theta$' becomes the slope as slope is

$$\frac{dy}{dx} = \tan\theta = \theta.$$

Now

$$AC = BC = \frac{L}{2}$$

Also from geometry of a circle, we know that

$$AC \times CB = DC \times CC'$$ $[\because DC = DC' - CC' = 2R - y]$.

$$\frac{L}{2} \times \frac{L}{2} = (2R-y) \times y$$

$$\frac{L^2}{4} = 2Ry - y^2$$

$\because$ for a practical beam, the deflection 'y' is so small, let us take $y^2 = 0$

$$\frac{L^2}{4} = 2Ry \Rightarrow y = \frac{L^2}{8R}$$

But from Bending moment equation, we know

$$\frac{M}{I} = \frac{E}{R} \Rightarrow R = \frac{E \times I}{M}$$

Substituting 'R' in 'y'.

$$y = \frac{L^2}{8\frac{EI}{M}}$$

$$* \boxed{y = \frac{ML^2}{8EI}} *$$

The equation gives the Central deflection of the beam which bends into a circular arc.

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Value of slope 'θ'

from the $\Delta^{le}$ AOB, we know that

$$\sin\theta = \frac{AC}{AD} = \frac{\left(\frac{L}{2}\right)}{R} = \frac{L}{2R}$$

Since the angle 'θ' is very small, hence $\sin\theta = \theta$

$$\theta = \frac{L}{2R} \Rightarrow \theta = \frac{L}{2\times\frac{EI}{M}}$$

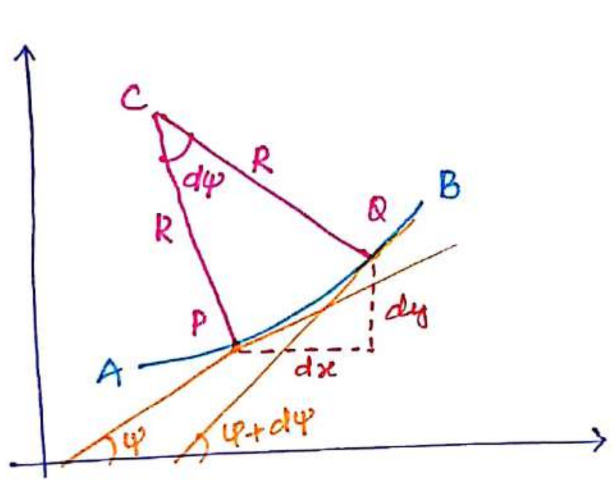
$$* \boxed{\theta = \frac{M\times L}{2EI.}} *$$

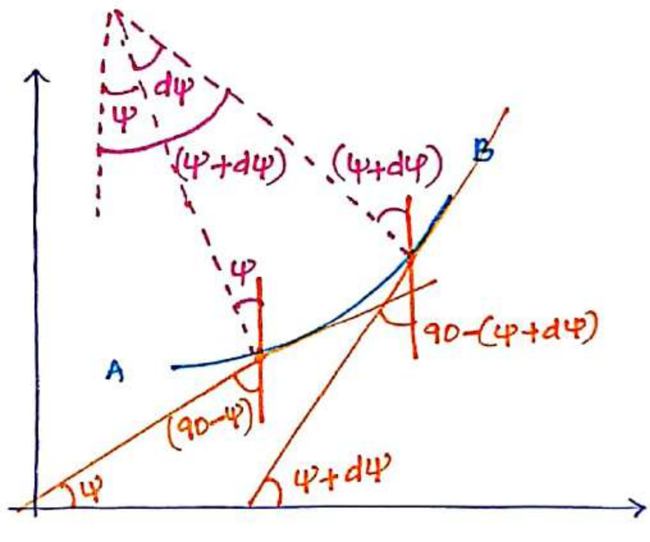
Equation gives the slope of the deflected beam at A & B.

## 2. RELATION B/W SLOPE, DEFLECTION & RADIUS OF CURVATURE:

Let the curve AB represents the deflection of a beam

Consider a small portion PQ of the beam, Let the tangents P and Q make angle $\psi$ and $\psi+d\psi$ with x-axis.



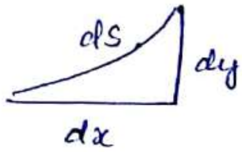

Normal at P and Q will meet at 'C', where 'C' is centre of curvature of 'PQ'.

Let length of PQ = 'ds'.

$\angle PCQ = d\psi$

$$PQ = ds = R\cdot d\psi \Rightarrow R = \frac{ds}{d\psi}$$

$$\tan\psi = \frac{dy}{dx}$$




$$\sin\psi = \frac{dy}{ds}$$

$$\cos\psi = \frac{dx}{ds}.$$

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As we know

$$R = \frac{ds}{d\psi} \Rightarrow \frac{\left(\frac{ds}{dx}\right)}{\left(\frac{d\psi}{dx}\right)} = \frac{\frac{1}{\cos\psi}}{\left(\frac{d\psi}{dx}\right)}$$

$$\Rightarrow \boxed{R = \frac{\sec\psi}{\left(\frac{d\psi}{dx}\right)}}$$

Differentiate '$\tan\psi$' with respect to '$x$'

$$\tan\psi = \frac{dy}{dx}$$

$$\sec^2\psi \cdot \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{d\psi}{dx} = \frac{(d^2y/dx^2)}{\sec^2\psi}$$

Substituting $\frac{d\psi}{dx}$ in '$R$'

$$R = \frac{\sec\psi}{\frac{(d^2y/dx^2)}{\sec^2\psi}}$$

$$R = \frac{\sec^3\psi}{(d^2y/dx^2)}$$

Taking Reciprocal on both sides.

$$\frac{1}{R} = \frac{(d^2y/dx^2)}{\sec^3\psi} = \frac{(d^2y/dx^2)}{(\sec^2\psi)^{3/2}}$$

$$= \frac{d^2y/dx^2}{(1+\tan^2\psi)^{3/2}}$$ [using trigonometric identity].

for a practical beam, slope '$\tan\psi$' is a small quantity,

Hence $\tan^2\psi$ can be neglected.

$$\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}.}$$

from the Bending equation, we have

$$\frac{M}{I} = \frac{E}{R}$$

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$$\boxed{\frac{1}{R} = \frac{M}{EI}}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \Rightarrow M = EI\frac{d^2y}{dx^2}$$

Differentiate w.r.to 'x'

$$\frac{dM}{dx} = EI \cdot \frac{d^3y}{dx^3} = \text{Shear force}$$

$$\boxed{F = EI \cdot \frac{d^3y}{dx^3}}$$

Differentiate w.r.to 'x'

$$\frac{dF}{dx} = EI \cdot \frac{d^4y}{dx^4} = \text{rate of loading}$$

$$\boxed{w = EI \cdot \frac{d^4y}{dx^4}}$$

Hence, the relation between curvature, slope, deflection, etc at any section given by.

* Deflection $= y$.
* Slope $= \frac{dy}{dx}$
* Bending moment $= EI\frac{d^2y}{dx^2}$
* Shear force $= EI\frac{d^3y}{dx^3}$
* rate of loading $= EI\frac{d^4y}{dx^4}$.

## 3. Methods of Determining Slope and Deflection:-

i, Double integration method

ii, Moment area method.

iii, Macaulay's method.

First two methods are used for a single load whereas the third one is used for several loads.

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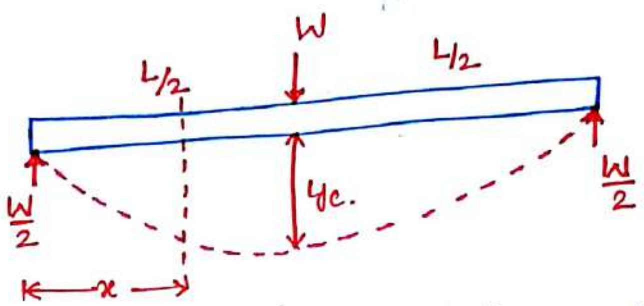
# 4. DEFLECTION OF A SIMPLY SUPPORTED BEAM WITH A POINT LOAD AT THE CENTRE:

Consider a S.S. beam AB of length 'L' and carrying a point load 'W' at the centre.




Reactions at Supports are $R_A$, $R_B$ ; $R_A = R_B = \frac{W}{2}$

Now

Consider a section 'X' at a distance 'x' from 'A'.

$$B.M @ x \Rightarrow M_x = R_A \cdot x$$

$$= \frac{W}{2} \times x. \quad \text{— i,}$$

But Bending moment at any section is given by

$$M = EI \frac{d^2y}{dx^2} \quad \text{— ii,}$$

Equating i, & ii, $\quad EI \frac{d^2y}{dx^2} = \frac{Wx}{2}$

On integrating we get

$$\text{Slope} = \boxed{EI \frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1}$$

where $C_1$ is the constant of integration. its value is obtained from boundary conditions.

$$@ \; x = \frac{L}{2} \; ; \quad \frac{dy}{dx} = 0$$

$$EI(0) = \frac{W}{2} \times \frac{x^2}{2} + C_1$$

$$\Rightarrow C_1 = -\frac{W}{2 \times 2}\left(\frac{L}{2}\right)^2$$

$$\Rightarrow \boxed{C_1 = -\frac{WL^2}{16}}$$

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Substituting the value of $C_1$ in equation, we get

$$\boxed{EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}}$$

It is the slope equation, slope is maximum at 'A'. At A, $x=0$ and hence slope at A is

$$EI\left(\frac{dy}{dx}\right)_{at\,A} = \frac{W}{4}(0) - \frac{WL^2}{16}$$

$$EI \cdot \theta_A = -\frac{WL^2}{16}$$

$$\boxed{\theta_A = -\frac{WL^2}{16EI}}$$

Deflection at a point is obtained by integrating the slope

$$\boxed{EI \times y = \frac{W}{4} \cdot \frac{x^3}{3} - \frac{WL^2}{16}x + C_2}$$

$C_2$ is another constant of integration, At A, $x=0$; deflection $=0$

$$EI \times 0 = \frac{W}{4} \cdot \left(\frac{0}{3}\right) - \frac{WL^2}{16}(0) + C_2$$

$$\boxed{C_2 = 0.}$$

$$\boxed{EI \times y = \frac{Wx^3}{12} - \frac{WL^2}{16}x}$$

The maximum deflection at centre, $x = \frac{L}{2}$

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16}\left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{12 \times 8} - \frac{WL^3}{16 \times 2}$$

$$= \frac{WL^3 - 3WL^3}{96} = -\frac{2WL^3}{96}$$

$$\boxed{y_c = -\frac{WL^3}{48EI}}$$

(Negative sign shows deflection is downwards).

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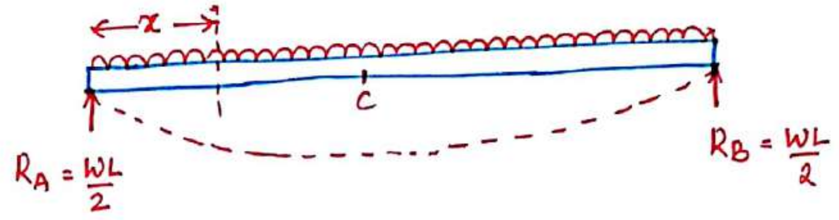
## 5. DEFLECTION OF A SIMPLY SUPPORTED BEAM WITH A UNIFORMLY DISTRIBUTED LOAD:




$$R_A = R_B = \frac{wL}{2}$$

Consider a section 'X' at a distance 'x' from 'A'

BM @ $x$ ; $M_x = R_A \times x - w \times x \times \frac{x}{2}$

$$M_x = \frac{wL}{2}x - \frac{w \cdot x^2}{2} \quad \text{— i,}$$

B·M @ any section is given by.

$$M = EI \frac{d^2y}{dx^2} \quad \text{— ii,}$$

equating i, & ii,

$$EI \cdot \frac{d^2y}{dx^2} = \frac{wL}{2}x - \frac{w \cdot x^2}{2}$$

Integrating the above, we get

$$\boxed{EI \cdot \frac{dy}{dx} = \frac{wL}{2} \cdot \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3} + C_1}$$

again by integrating

$$\boxed{EI \cdot y = \frac{wL}{4} \frac{x^3}{3} - \frac{w}{6} \cdot \frac{x^4}{4} + C_1 x + C_2}$$

The integral constants $C_1$ & $C_2$ are obtained by boundary conditions

i, at $x = 0$; $y = 0$. ii, at $x = L$; $y = 0$

$$EI(0) = \frac{wL}{4}(0) - \frac{w}{6}(0) + C_1(0) + C_2$$

$$\Rightarrow \boxed{C_2 = 0}$$

$$EI(0) = \frac{wL}{12}(L^3) - \frac{w}{24}(L^4) + C_1 L + 0$$

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$$0 = \frac{wL^4}{12} - \frac{wL^4}{24} + C_1 L$$

$$C_1 = -\frac{wL^3}{12} + \frac{wL^3}{24}$$

$$\boxed{C_1 = -\frac{wL^3}{24}}$$

Substituting $C_1$ & $C_2$

$$\boxed{EI\frac{dy}{dx} = \frac{w \cdot L}{4}x^2 - \frac{w \cdot}{6}x^3 - \frac{wL^3}{24}.}$$

$$\boxed{EI\, y = \frac{wL}{12}x^3 - \frac{w}{24}x^4 - \frac{wL^3}{24}x.}$$

Slope at supports

Let $\theta_A$ is slope at 'A', $x=0$; $\frac{dy}{dx} = \theta_A$

$$EI \times \theta_A = \frac{w \cdot L}{4}(0) - \frac{w}{6}(0) - \frac{wL^3}{24}$$

$$\boxed{\theta_A = -\frac{wL^3}{24EI}}$$

By Symmetry

$$\boxed{\theta_B = -\frac{wL^3}{24EI}}$$

(or) $\boxed{\theta_A \text{ or } \theta_B = -\frac{WL^2}{24EI}}$

maximum deflection

maximum deflection occurs at centre of beam, $x = \frac{L}{2}$

$$EI \cdot y_c = \frac{wL}{12}\left[\frac{L}{2}\right]^3 - \frac{w}{24}\left[\frac{L}{2}\right]^4 - \frac{wL^3}{24}\left[\frac{L}{2}\right]$$

$$= \frac{wL^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48}$$

$$= -\frac{5wL^4}{384}$$

* $\boxed{y_c = -\frac{5}{384}\frac{wL^4}{EI}.}$ * (or) $\boxed{y_c = -\frac{5}{384}\frac{WL^3}{EI}.}$

Negative Sign indicates deflection is downwards.

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**Q:** A beam of uniform rectangular section 200 mm wide and 300 mm deep is S.S at ends. It carries a U.D.L of 9 KN/m run over the entire span of 5m. If the value of E for the beam material is $1 \times 10^4$ N/mm², find.

i, The slope at the supports

ii, Maximum deflection.

**Q:** A beam of length 5m and of U.D.L of 9 KN/m over entire length having a uniform rectangular section supported at its ends. Calculate the width and depth of the beam if permissible bending stress is 7 N/mm² and central deflection is not to exceed 1cm. take $E = 1 \times 10^4$ N/mm²

**Sol:** Given

Length, $L = 5m = 5000$ mm

U.D.L, $w = 9$ KN/m.

Total load, $W = w.L = 9 \times 5 = 45$ KN $= 45000$N.

Bending stress, $f = 7$ N/mm².

Central deflection, $y_c = 1cm = 10mm$.

$E = 1 \times 10^4$ N/mm².

Let the dimensions be $b \times d$.

$MOI = \frac{bd^3}{12}$

We know $y_c = \frac{5}{384} \frac{W \cdot L^3}{EI}$

$$10 = \frac{5}{384} \times \frac{45000 \times 5000^3}{1 \times 10^4 \times \frac{bd^3}{12}}$$

$$bd^3 = 878.906 \times 10^7 \text{ mm}^4.$$

max. B.M $= \frac{wL^2}{8} = \frac{WL}{8} = \frac{45000 \times 5}{8} \times 1000 = 28125000$ Nmm

Now $\frac{M}{I} = \frac{f}{y} \Rightarrow \frac{28125000}{\frac{bd^3}{12}} = \frac{7}{(d/2)} \Rightarrow bd^2 = 24107142.85 \text{ mm}^3$.

$$\Rightarrow d = \frac{878.906 \times 10^7}{24107142.85}$$

$d = 364.58$ mm.

$b \times (364.58)^2 = 24107142.85$

$b = 181.36$ mm,

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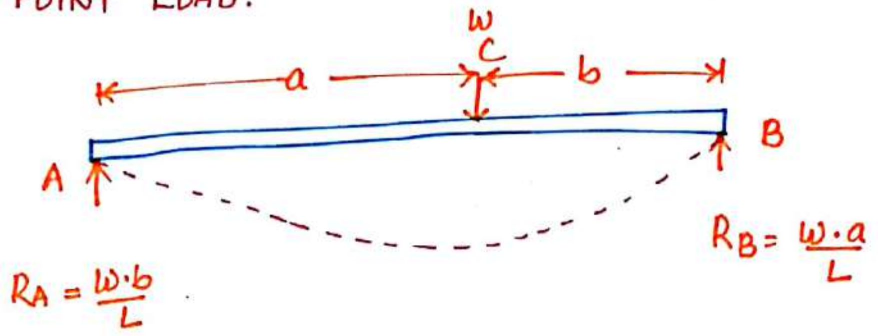
# 6. MACAULAY'S METHOD :-

DEFLECTION OF A SIMPLY SUPPORTED BEAM WITH AN ECCENTRIC POINT LOAD:




Bending moment at any section b/w A & C . at a distance 'x' from 'A'

$$M_x = R_A \times x$$

$$= \frac{W \cdot b}{L} \times x .$$

Bending moment at any section b/w C & B

$$M_x = \frac{W \cdot b}{L} \times x - W(x-a) .$$

The B.M for all the sections of the beam can be expressed in single equation as

$$M_x = \frac{W \cdot b}{L} \times x \;\vdots\; - W(x-a) .$$

The B.M at any section is given by $M = EI \frac{d^2y}{dx^2}$

$$EI \frac{d^2y}{dx^2} = \frac{W \cdot b}{L} \times x \;\vdots\; - W(x-a)$$

Integrating the above equation , we get

$$EI \frac{dy}{dx} = \frac{W \cdot b}{L} \cdot \frac{x^2}{2} + C_1 \;\vdots\; - \frac{W(x-a)^2}{2}$$

Integrating above equation once again , we get

$$EI\, y = \frac{W \cdot b}{2L} \frac{x^3}{3} + C_1 x + C_2 \;\vdots\; - \frac{W}{2} \frac{(x-a)^3}{3}$$

The values of $C_1$ & $C_2$ are obtained from boundary conditions

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The two boundary conditions are

i) At $x=0$ ; $y=0$ ii) At $x=L$ ; $y=0$

i) at $x=0$ ; $y=0$

$$\Rightarrow \quad EI(0) = \frac{W \cdot b}{2L}(0) + C_1(0) + C_2$$

$$\boxed{C_2 = 0}$$

ii) at B, $x=L$ ; $y=0$

$$EI(0) = \frac{W \cdot b}{2L} \cdot \frac{L^3}{3} + C_1 \cdot (L) + 0 - \frac{W}{2}\frac{(L-a)^3}{3}$$

$$= \frac{WbL^2}{6} + C_1 L - \frac{Wb^3}{6}$$

$$C_1 L = \frac{W}{6}b^3 - \frac{WbL^2}{6} = -\frac{Wb}{6}\left[L^2 - b^2\right]$$

$$\boxed{C_1 = -\frac{Wb}{6L}\left[L^2 - b^2\right]}$$

Substituting the value of $C_1$ in slope we get

$$EI\frac{dy}{dx} = \frac{W \cdot b \cdot x^2}{2L} + \left[-\frac{Wb}{6L}(L^2 - b^2)\right] \; \vdots \; - \frac{W(x-a)^2}{2}$$

$$\boxed{EI\,\theta = \frac{W \cdot b \cdot x^2}{2L} - \frac{W \cdot b}{6L}(L^2 - b^2) \; \vdots \; - \frac{W(x-a)^2}{2}}$$

The above equation gives slope at any point in the beam.

Slope at A ; $x=0$

$$EI\,\theta_A = \frac{W \cdot b}{2L}(0) - \frac{W \cdot b}{6L}(L^2 - b^2)$$

$$= -\frac{W \cdot b}{6L}(L^2 - b^2)$$

$$\boxed{\theta_A = -\frac{Wb}{6EIL}(L^2 - b^2)}$$

Slope at B; $x=L$

$$EI\,\theta_B = \frac{W \cdot b}{2L}L^2 - \frac{Wb}{6L}(L^2 - b^2) - \frac{W(L-a)^2}{2}$$

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$$EI\,\theta_B = \frac{WbL}{2} - \frac{Wb}{6L}(L^2-b^2) - \frac{W(b)^2}{2}$$

$$\theta_B = \frac{Wb}{2EI}\left[L - \frac{(L^2-b^2)}{3L} - b\right].$$

$$\theta_B = \frac{Wb}{2EI}\left[\frac{3aL-(L^2-b^2)}{3L}\right]$$

$$\boxed{\theta_B = \frac{Wb}{6EIL}\left[3a^2+3ab-L^2+b^2\right]}$$

Substitute the values of $C_1$ & $C_2$ in deflection, we get

$$\boxed{EI\cdot y = \frac{W\cdot b}{6L}\cdot x^3 + \left[-\frac{Wb}{6L}(L^2-b^2)\right]x + 0 \;\vdots\; -\frac{W}{6}(x-a)^3}$$

equation gives the deflection at any point in the beam.

To find the deflection at 'c' ; $x = a$

$$EI\cdot y_c = \frac{W\cdot b}{6L}(a)^3 - \frac{Wb}{6L}(L^2-b^2)(a)$$

$$= \frac{W\cdot b}{6L}\cdot a\left[a^2 - L^2 + b^2\right]$$

$$= -\frac{W\cdot b\cdot a}{6L}\left[L^2-a^2-b^2\right].$$

$$= -\frac{W\cdot a\cdot b}{6L}\left[(a+b)^2-a^2-b^2\right]$$

$$= -\frac{W\cdot a\cdot b}{6L}\left[a^2+b^2+2ab-a^2-b^2\right]$$

$$= -\frac{W\,a\cdot b}{6L}\times 2\cdot a\cdot b$$

$$EI\,y_c = -\frac{W\,a^2b^2}{3L}$$

$$\boxed{y_c = -\frac{W\,a^2b^2}{3EIL}}$$

*

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**Q:** A beam of length 6m is supported simply at its ends and carries two points loads of 48 KN and 40 KN. at a distance of 1m and 3m respectively from the left support. find.

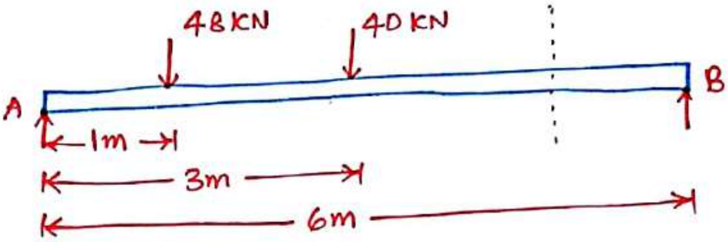
i, deflection under each load

ii, maximum deflection, and

iii, the point at which maximum deflection occurs.

Given, $E = 2 \times 10^5 \ N/mm^2$ and $I = 85 \times 10^6 \ mm^4$.

**Sol:** Given




$I = 85 \times 10^6 \ mm^4$

$E = 2 \times 10^5 \ N/mm^2$.

Calculate the reactions ~~and~~ at $R_A$ & $R_B$.

Taking moments about 'A'

$$R_B \times 6 = 48 \times 1 + 40 \times 3 = 168$$

$$R_B = \frac{168}{6} = 28 \ KN.$$

$$R_A = \text{Total load} - R_B = (48+40) - 28 = 60 \ KN.$$

Consider the section 'X' in the last part of the beam at a distance 'x'.

BM at this section is given by.

$$EI \frac{d^2y}{dx^2} = R_A \cdot x \ \vdots \ -48(x-1) \ \vdots \ -40(x-3)$$

$$= 60x \ \vdots \ -48(x-1) \ \vdots \ -40(x-3)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = 60\frac{x^2}{2} + C_1 \ \vdots \ -48\frac{(x-1)^2}{2} \ \vdots \ -40\frac{(x-3)^2}{2}$$

$$= 30x^2 + C_1 \ \vdots \ -24(x-1)^2 \ \vdots \ -20(x-3)^2$$

Integrating the above equation again, we get.

$$EI.\,y = \frac{30x^3}{3} + C_1 x + C_2 \ \vdots \ -24\frac{(x-1)^3}{3} \ \vdots \ -20\frac{(x-3)^3}{3}$$

$$= 10x^3 + C_1 x + C_2 \ \vdots \ -8(x-1)^3 \ \vdots \ -\frac{20}{3}(x-3)^3.$$

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To find the value of $C_1$ & $C_2$, use two boundary conditions

i, at $x = 0$ ; $y = 0$ and ii, at $x = 6m$ ; $y = 0$

Substituting first boundary conditions

$$0 = 0 + 0 + C_2 \Rightarrow \therefore \boxed{C_2 = 0.}$$

Substituting second boundary condition:

$$0 = 10 \times 6^3 + C_1 \times 6 + 0 - 8(6-1)^3 - \frac{20}{3}(6-3)^3$$

$$0 = 2160 + 6C_1 - 8 \times 5^3 - \frac{20}{3} \times 3^3$$

$$= 980 + 6C_1$$

$$C_1 = \frac{-980}{6} \Rightarrow \boxed{C_1 = -163.33}$$

Now Substituting the values of $C_1$ & $C_2$

$$\boxed{EI\,y = 10x^3 - 163.33x \; \vdots \; -8(x-1)^3 \; \vdots \; -\frac{20}{3}(x-3)^3.}$$

i, Deflection under first load:

at point C, obtained by substituting $x = 1m$ upto first dotted line

$$EI \cdot y_C = 10 \times 1^3 - 163.33 \times 1$$

$$= -153.33 \text{ kNm}^3$$

$$= -153.33 \times 10^3 \times 10^9 \text{ Nmm}^3$$

$$y_C = \frac{-153.33 \times 10^{12}}{EI}$$

$$= \frac{-153.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} \text{ mm}$$

$$\boxed{y_C = -9.019 \text{ mm}}$$ (-ve represents downward deflection).

ii, Deflection under second load:

at point D, obtained by substituting $x = 3m$

$$EI \cdot y_D = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)^3$$

$$= 270 - 489.99 - 64$$

$$= -283.99 \times 10^{12} \text{ Nmm}^3$$

$$y_D = \frac{-283.99 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$\boxed{y_D = -16.7 \text{ mm}}$$

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iii) maximum deflection:

The deflection is likely to be maximum at sctn b/w C & D.

for max. deflection, $\frac{dy}{dx} = 0$.

$$30x^2 + C_1 - 24(x-1)^2 = 0$$

$$30x^2 - 163.33 - 24(x^2 + 1 - 2x) = 0$$

$$6x^2 + 48x - 187.33 = 0.$$

Solution for above quadratic equation

$$x = \frac{-48 \pm \sqrt{48^2 + 4 \times 6 \times 187.33}}{2 \times 6}$$

$$= 2.87\ m.$$

Substituting $x = 2.87\,m$ in equation for max 'y'

$$EI\, y_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87-1)^3$$

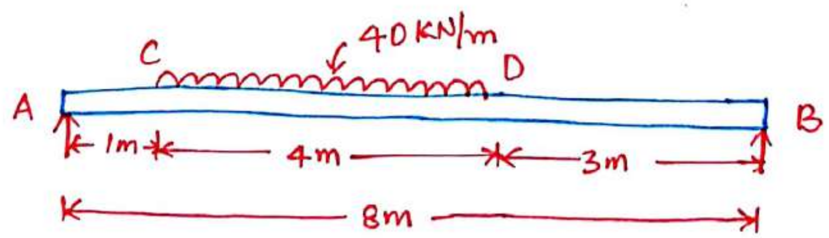
$$= 236.39 - 468.75 - 52.31$$

$$= -284.67 \times 10^{12}\ Nmm^3$$

$$y_{max} = \frac{-284.67 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$\boxed{y_{max} = -16.745\ mm}$$

Q: A beam of length 8m is simply supported at its ends. It carries a U.D.L of 40 KN/m as shown in figure. Determine the deflection of the beam at its mid span and also the position of maximum deflection and maximum deflection. Take $E = 2 \times 10^5\ N/mm^2$ and $I = 4.3 \times 10^8\ mm^4$

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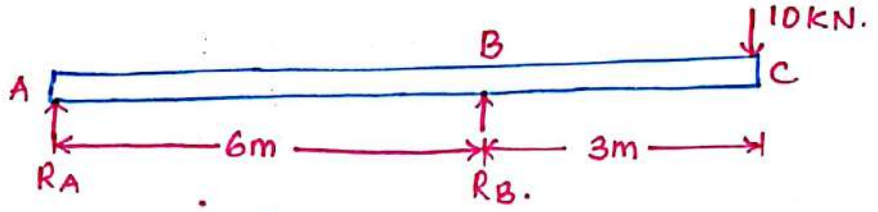
Q An Over hanging beam ABC is loaded as shown in figure. Find the Slopes over each Support and at the right end. Find also the maximum upward deflection between the Supports and the deflection at the right end.

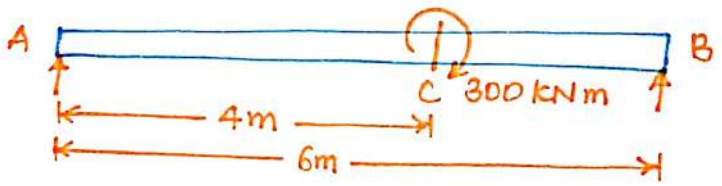
Take $E = 2 \times 10^5$ N/mm$^2$ and $I = 5 \times 10^8$ mm$^4$




Q: A Horizontal beam AB is Simply Supported at A and B, 6m apart The beam is Subjected to a Clockwise Couple of 300 KNm at a distance of 4m from the left end as shown in figure. If $E = 2 \times 10^5$ N/mm$^2$ and $I = 2 \times 10^8$ mm$^4$, determine.

i, deflection at the point where Couple is acting.

ii, The maximum deflection.

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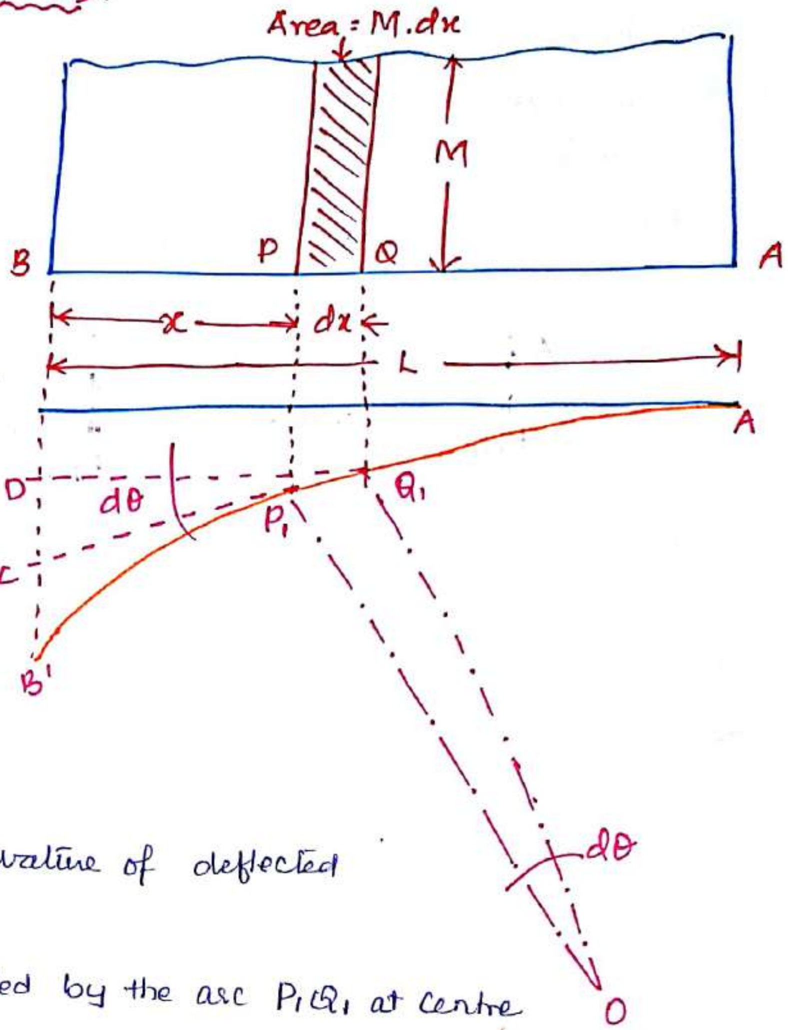
# 7. Moment Area Method:-




Consider a beam AB carrying some loading

Let the beam bent into $AQ_1P_1B$ as shown.

Due to the load acting on beam. let A be the point of zero slope and zero deflection.

Consider a element PQ of small length 'dx' at a distance 'x' from B.

Let R – Radius of curvature of deflected part $P_1Q_1$

dθ – Angle subtended by the arc $P_1Q_1$ at centre

M – Bending moment between P and Q.

$P_1C$ – Tangent at point $P_1$

$Q_1D$ – Tangent at point $Q_1$.

The angle b/w lines $CP_1$ and $DQ_1$ will be equal to 'dθ'

for the deflected part $P_1Q_1$ of beam we have

$$P_1Q_1 = R \cdot d\theta$$

$$P_1Q_1 \approx dx$$

$$dx = R \cdot d\theta$$

$$\boxed{d\theta = \frac{dx}{R}} \quad \text{—i,}$$

But we know

$$\frac{M}{I} = \frac{E}{R} \quad \Rightarrow \boxed{R = \frac{EI}{M}}$$

Substitute 'R' in dθ we get

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$$d\theta = \frac{dx}{\left(\frac{EI}{M}\right)} = \frac{M.dx}{EI.}$$

Since the slope at a point 'A' is assumed zero, total slope at 'B' is obtained by integrating the above equation between the limits 0 & L.

$$\theta = \int_0^L \frac{M.dx}{EI} = \frac{1}{EI}\int_0^L M\,dx.$$

'M' represents the area of B.M diagram of length 'dx'

Hence $\int_0^L M.dx$ represents the area of B.M diagram b/w A & B.

$$\theta = \frac{1}{EI} \times \text{Area of B.M diagram b/w A \& B.}$$

∴ Slope at 'B'

$$\boxed{\theta_B = \frac{\text{Area of B.M diagram b/w A \& B}}{EI.}}$$

Now the deflection due to Bending of the portion $P_1Q_1$ is given by

$$dy = x.d\theta$$

$$dy = x.\frac{M.dx}{EI.}$$

Since deflection @A is zero, hence the total deflection at 'B' is obtained by integrating the above equation b/w limits 0 to L.

$$y = \int_0^L \frac{x M.dx}{EI}$$

$$= \frac{1}{EI}\int_0^L x.M\,dx.$$

But $x \times M\,dx$ represents the moment of area of the B.M diagram of length 'dx' about point 'B'.

This is equal to the total area of B.M multiplied by the distance of C.G of the B.M diagram

$$\boxed{y = \frac{1}{EI} \times A \times \bar{x}}$$

A - Area of B.M diagram b/w A & B.

$\bar{x}$ - Distance of C.G of Area from A to B.

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## 8. MOHR'S THEOREMS :-

The results given for slope and deflection in moment area method are known as "Mohr's theorems"

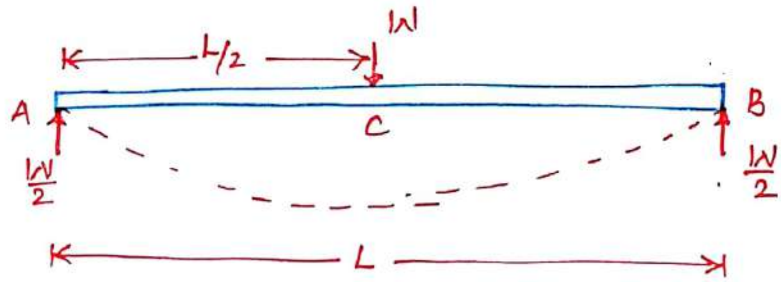
I. The change of slope between any two points is equal to the net area of the B.M diagram between these points divided by EI.

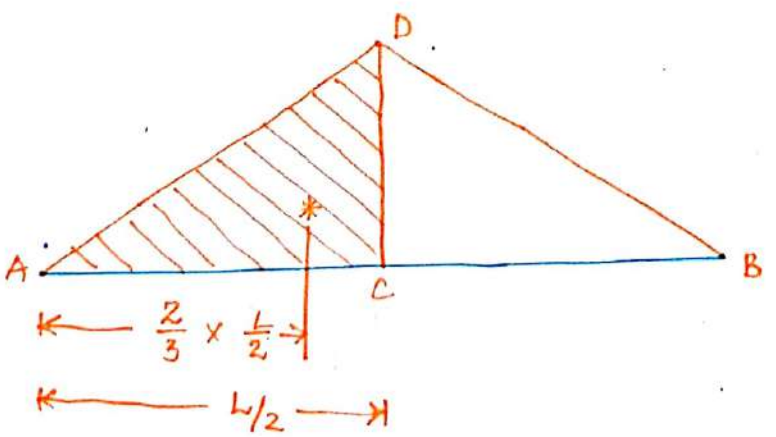
II. The total deflection between any two points is equal to the moment of the area of BM diagram between the two points about the last point divided by EI.

The Mohr's theorems is conveniently used for following cases

1. Problems on cantilevers (Zero slope at fixed end).
2. Simply supported beams carrying symmetrical loading (Zero slope at the centre).
3. Beams fixed at both ends (Zero slope at each end).

### ⊛ SLOPE & DEFLECTION OF A SIMPLY SUPPORTED BEAM CARRYING A POINT LOAD AT THE CENTRE BY MOHR's THEOREM:



Consider a Simply Supported beam AB of length 'L' and Carrying a point load 'W' at the Centre of the beam i.e., at 'C'.

∵ This is a case of Symmetrical loading, hence slope is zero at the centre.

Now, using mohr's theorem for slope

Slope at 'A'

$$A = \frac{\text{Area of B.M diagram b/w A \& C}}{EI}$$

Area of B.M diagram b/w A & C

$= \text{Area of } \Delta^{le} ACD$

$= \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4} = \frac{WL^2}{16}$

∴ Slope at A is $\boxed{\theta_A = \frac{WL^2}{16EI}}$

Now using Mohr's theorem for deflection, we get

$$y = \frac{A\bar{x}}{EI}.$$

A - Area of BM diagram b/w A & C

$= \frac{WL^2}{16}$

$\bar{x}$ - Distance of C.G of area 'A' from A

$= \frac{2}{3} \times \frac{L}{2} = \frac{L}{3}$

$$\therefore y = \frac{\frac{WL^2}{16} \times \frac{L}{3}}{EI}$$

$$\boxed{y = \frac{WL^3}{48EI}.}$$

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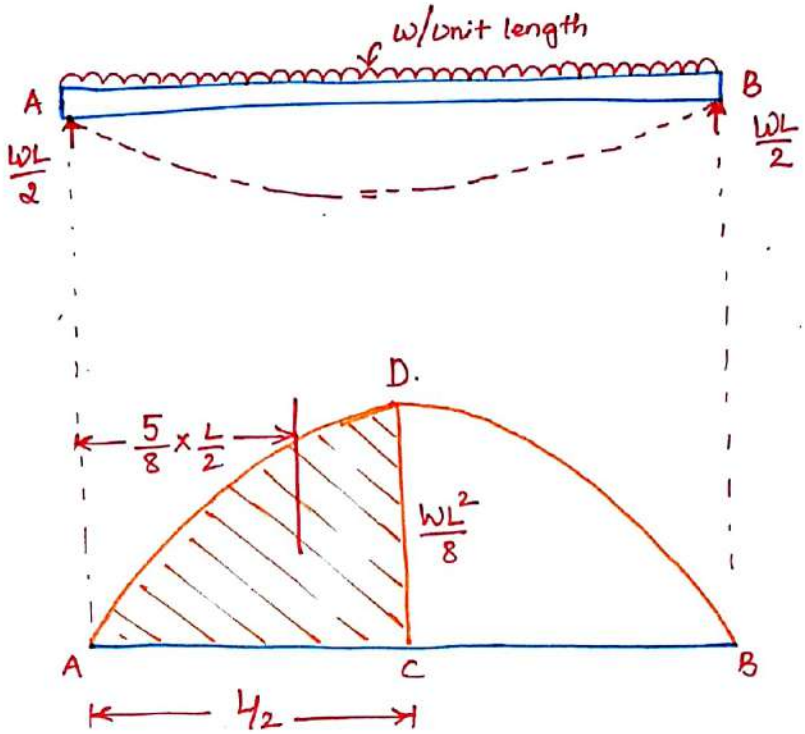
# * SLOPE & DEFLECTION OF A SIMPLY SUPPORTED BEAM CARRYING U.D.L BY MOHR'S THEOREM:




Slope at A = $\dfrac{\text{Area of B.M diagram b/w A and C.}}{EI}$

Area = Area of parabola ACD

$= \frac{2}{3} \times AC \times CD = \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8} = \frac{w \cdot L^3}{24}$

$\therefore \boxed{\theta_A = \frac{wL^3}{24EI}}$

for deflection $\bar{x}$ = Distance of C.G of area

$= \frac{5}{8} \times AC = \frac{5}{8} \times \frac{L}{2} = \frac{5L}{16}$

$y = \dfrac{\frac{wL^3}{24} \times \frac{5L}{16}}{EI}$ $\boxed{y = \frac{5}{384} \frac{w \cdot L^4}{EI}.}$

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# 9. CONJUGATE BEAM METHOD:-

The slope and deflections of beams and cantilevers obtained from various methods become laborious, when applied to beams whose flexural rigidity is not uniform throughout the length of the beam.

The slopes and deflections of such beams can be easily obtained by Conjugate beam method.

## CONJUGATE BEAM:-

It is an imaginary beam of length equal to that of the original beam but for which the load diagram is the $\frac{M}{EI}$ diagram.

[The load at any point on the conjugate beam is equal to BM at that point divided by EI].

*1. The Slope at any section of the given beam is equal to the "Shear force" at the corresponding section of the "Conjugate beam".

*2. The deflection at any section for the given beam is equal to the "Bending moment" at the corresponding section of the "Conjugate beam".

# 10. SLOPE & DEFLECTION OF A S.S. BEAM WITH A POINT LOAD AT CENTRE:

(•) Consider a simply supported beam AB of length 'L' carrying a point load 'W' at the centre 'C'

(•) The B·M at A and B is Zero and at the Centre B·M will be $\frac{W \cdot L}{4}$.

(•) The B·M varies according to Straight line law.

(•) The load on the conjugate beam will be obtained by dividing the B·M at that point by EI.

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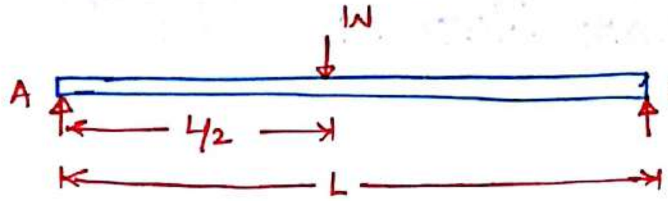


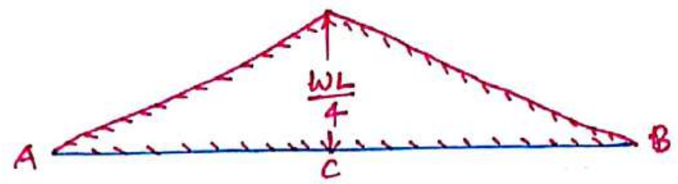


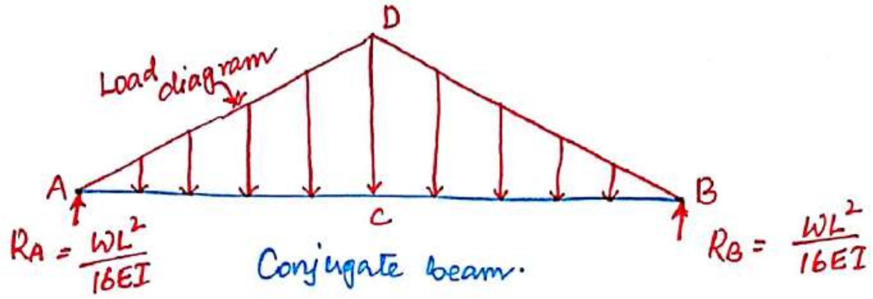


Total load on the conjugate beam = Area of the load diagram

$$= \frac{1}{2} \times AB \times CD$$

$$= \frac{1}{2} \times L \times \frac{WL}{4EI}.$$

$$= \frac{WL^2}{8EI}$$

Reaction at each support for the conjugate beam will be half of the total load.

$$R_A^* = R_B^* = \frac{1}{2} \frac{WL^2}{8EI} = \frac{WL^2}{16EI}$$

According to Conjugate Beam method:

$\theta_A$ = Shear force at 'A' for the conjugate beam.

$$= R_A^*$$

$$\boxed{\theta_A = \frac{WL^2}{16EI}}$$

$y_c$ = B.M at C for the conjugate beam

$= R_A^* \times \frac{L}{2}$ - Load Corresponding to ACD.

$$= \frac{WL^2}{16EI} \times \frac{L}{2} - \left(\frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI}\right) \times \left(\frac{L}{2} \times \frac{1}{3}\right)$$

$$\boxed{y_c = \frac{WL^3}{48EI}}$$

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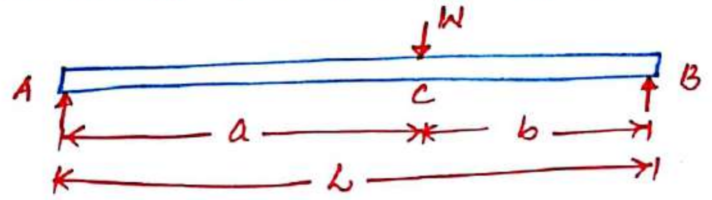
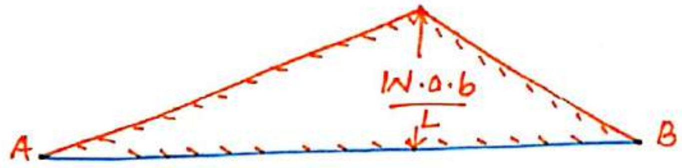
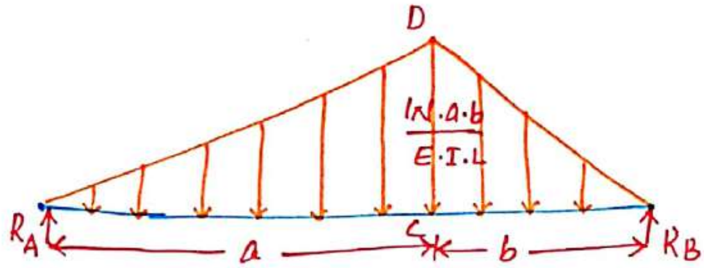
## 11. SIMPLY SUPPORTED BEAM CARRYING AN ECCENTRIC LOAD:




Reactions acting at A and B are

$$R_A = \frac{W \cdot b}{L} \quad ; \quad R_B = \frac{W \cdot a}{L}$$

The B·M at A and B will be zero; BM at 'C' will be $R_A \times a$

$$= \frac{W \times b}{L} \times a = \frac{W \cdot a \cdot b}{L}$$

The vertical Load on conjugate beam at $C^*$ will be $\frac{W \cdot a \cdot b}{EI \cdot L}$

To find reactions at A and B,

Taking moments about A

$$R_B^* \cdot L = \left[\frac{1}{2} \times AC \times CD\right] \times \left[\frac{2}{3} \times AC\right] + \left[\frac{1}{2} \times BC \times CD\right]\left[AC + \frac{1}{3} CB\right]$$

$$= \frac{1}{2} \times a \times \frac{W \cdot ab}{EIL} \times \frac{2}{3} \times a + \frac{1}{2} \times b \times \frac{W \cdot a \cdot b}{EIL} \times \left(a + \frac{b}{3}\right)$$

$$= \frac{W \cdot a \cdot b}{2EIL}\left[a \times \frac{2}{3} \times a + ba + \frac{b^2}{3}\right]$$

$$= \frac{W \cdot a \cdot b}{2EIL}\left[\frac{2a^2 + 3ab + b^2}{3}\right]$$

$$= \frac{W \cdot a \cdot b}{6EIL}\left[(a+b)^2 + a^2 + ab\right]$$

$$= \frac{W \cdot a \cdot b}{6EIL}\left[L^2 + a(a+b)\right]$$

$$= \frac{W \cdot a \cdot b}{6EIL}\left[L^2 + aL\right]$$

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$$= \frac{W \cdot a \cdot b}{6EIL}\left[L^2 + a \cdot L\right]$$

$$R_B \times L = \frac{W \cdot a \cdot (L-a)}{6EI\cancel{L}} \times \cancel{L}(L+a)$$

$$\boxed{R_B = \frac{W \cdot a}{6EIL}\left[L^2 - a^2\right]}$$

Similarly $\boxed{R_A = \frac{W \cdot b}{6EIL}\left[L^2 - b^2\right]}$

let $\theta_A$ = Slope at A & $y_c$ - Deflection at 'c'

Now, According to Conjugate beam method

$\theta_A$ = Shear force at 'A' for the Conjugate beam

$= R_A$

$$* \boxed{\theta_A = \frac{W \cdot b}{6EIL}(L^2 - b^2)} *$$

$y_c$ = B.M at C for Conjugate beam

$= R_A^* \cdot a$ - Load ACD x Distance of C.G of ACD from C

$$= \frac{W \cdot b}{6EIL}(L^2 - b^2) \times a - \left(\frac{1}{2} \times a \times \frac{W \cdot a \cdot b}{EIL}\right) \times \left(\frac{a}{3}\right)$$

$$= \frac{W \cdot a \cdot b}{6EIL}(L^2 - b^2) - \frac{W \cdot a^3 \cdot b}{6EIL}$$

$$* \boxed{y_c = \frac{W \cdot a \cdot b}{6 \cdot EIL}\left[L^2 - b^2 - a^2\right].} *$$

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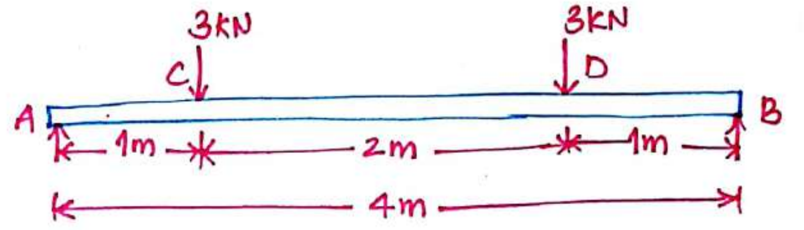
Q. A simply supported beam of length 4m carries a point load of 3KN at a distance of 1m from each end. If $E = 2\times10^5\ N/mm^2$ and $I = 10^8\ mm^4$ for the beam, then using conjugate beam method, determine

i, Slope at each end and under each load

ii, deflection under each load and at the centre.

Sol:- Given

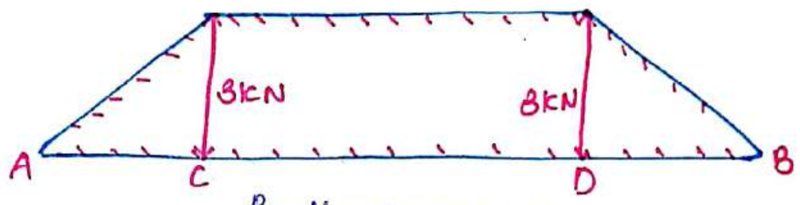



length, $L = 4m$

$E = 2\times10^5\ N/mm^2$

$= \frac{2\times10^5\times10^{-3}}{10^{-6}}\ \frac{KN}{m^2}$

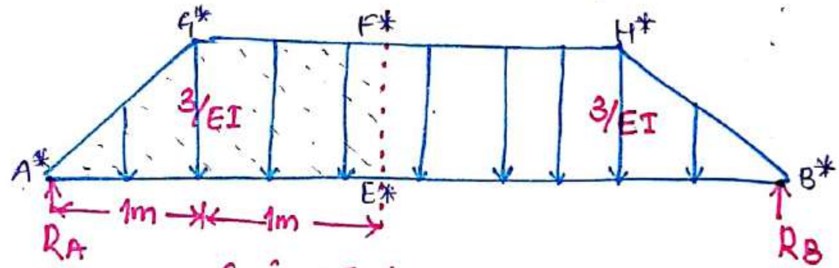
$= 2\times10^8\ KN/m^2$




Bending moment diagram.

$I = 10^8\ mm^4 = \frac{10^8}{10^{12}}\ m^4$

$= 10^{-4}\ m^4$




Conjugate beam.

Reactions at A & B are

$R_A = R_B = 3KN$

B.M @ A & B = 0

BM @ C = $R_A \times 1 = 3\times1 = 3KN$

BM @ D = $R_B\times1 = 3\times1 = 3KN$ (or), $R_A\times3 - R\times2 = 9\times3 - 3\times2 = 3\ KN$.

From the obtained Bending moment dividing at any section by EI, we can construct conjugate beam.

let $R_A^*$ & $R_B^*$ = Reactions of Conjugate Beam.

$R_A^* = R_B^*$ = Half of total load on conjugate Beam.

$= \frac{1}{2}\left[A^* B^* H^* G^*\right]$

$= \frac{1}{2}\left[\frac{(4+2)}{2} * \frac{3}{EI}\right]$

$R_A^* = R_B^* = \frac{4.5}{EI}$.

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i, Slope at each end and under each load:-

Slope at A, $\theta_A$ = Shear force at A for conjugate beam

$$= R_A^* = \frac{4.5}{EI} = \frac{4.5}{2\times10^8 \times 10^{-4}}$$

$$= 0.000225 \text{ rad.}$$

$$\theta_B = R_B^* = \frac{4.5}{EI} = 0.000225 \text{ rad}$$

$\theta_C$ = Shear force at 'C' for conjugate beam

$= R_A^*$ − Total load at $\Delta$le

$$= \frac{4.5}{EI} - \frac{1}{2}\times1\times\frac{3}{EI} = \frac{3}{EI} = \frac{3}{2\times10^8\times10^{-4}}$$

$$= 0.00015 \text{ rad.}$$

Similarly, $\theta_D = 0.00015$ rad.

ii, Deflection under each load:-

let $y_C$ & $y_D$ are deflections at 'C' & 'D' for the beam

Now according to conjugate beam

$y_C$ = B.M at $C^*$ for conjugate beam

$= R_A^* \times 1.$ − [Triangle load] × Distance of C.G

$$= \frac{4.5}{EI}\times1 - \left[\frac{1}{2}\times1\times\frac{3}{EI}\right]\times\frac{1}{3}$$

$$= \frac{4.0}{EI} = \frac{4.0}{2\times10^8\times10^{-4}} \text{ m} = \frac{4\times1000}{2\times10^4} \text{ mm}$$

$$= \underline{0.2 \text{ mm}}$$

Deflection at the centre of the beam

= B.M at the centre of the conjugate beam.

$= R_A^* \times 2$ − load $A^*E^*F^*$ × Distance of C.G

$$= \frac{4.5}{EI}\times2 - \left(\frac{1}{2}\times1\times\frac{3}{EI}\right)\times\left(1+\frac{1}{3}\right) - \left(1\times\frac{3}{EI}\right)\times\frac{1}{2}$$

$$= \frac{9}{EI} - \frac{2}{EI} - \frac{1.5}{EI} = \frac{6.5}{EI}$$

$$= \frac{6.5}{2\times10^8\times10^{-4}} \text{ m} = \frac{6.5\times1000}{2\times10^4} = \underline{0.325 \text{ mm}}$$

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