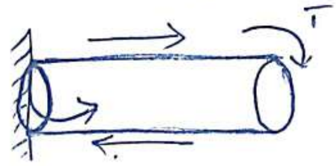
UNIT - I TORSION 11/02/25

Torsion :- Torsion is the effect of torque applied. It is the twisting of a straight bar when it is loaded by torque that tend to produce rotation about Longitudinal axis



Pure torsion :- when the cross sections are subjected to only torsional moments and not accompanied by any axial forces & Bending Moment then it is said to be in pure torsion.

Assumptions made in the torsion theory :-

1) Material of the shaft is uniform throughout
2) Twist along the shaft is uniform.
3) Shaft is uniform circular cross-section.
4) cross-section of the shaft which is plane before the twist is plane after the twist.
5) All radii which are straight before twist remains straight after the twist.

Derivation :-

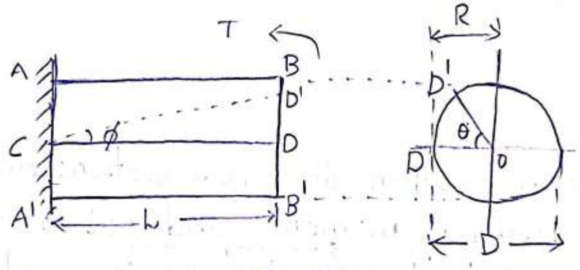
(A shaft) consider a shaft fixed at one end AA' and free at the other end BB' as shown in the figure.

→ Let CD be any line on the outer surface of the shaft.

Let shaft subjected to a torque 'T' at the end BB' and every cross section of the shaft will be subjected to shear stress.

Hence the point D will be shift to the D' and line CD will become CD' as shown in the figure and the line OD will be shifter to OD'

$OD = OD' = R$



R = Radius of the shaft

L = Length of the shaft

T = Torque applied at the end of the shaft

G = Modulus of rigidity

τ = shear stress induced at the surface of shaft due to Torque T

ϕ = angle DCD' = shear strain

θ = angle of twist = Angle DOD'

Distortion at the outer surface due to the torque

$T = DD'$

Therefore shear strain at outer surface $\tan\phi$ $\frac{DD'}{CD}$

$\tan\phi = \frac{DD'}{CD}$

$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$

$G = \frac{\tau}{\phi} \rightarrow ?$

$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$

from Δ^{le} DCD'

$\tan\phi = \frac{DD'}{CD} = \frac{DD'}{L}$

for small angles, $\tan\phi \simeq \phi$

$\phi = \frac{DD'}{L}$

DD' :-

Length of arc $= R\theta$

$DD' = R\theta$

$\phi = \frac{DD'}{L} = \frac{R\theta}{L}$

$G = \frac{\tau}{\phi}$

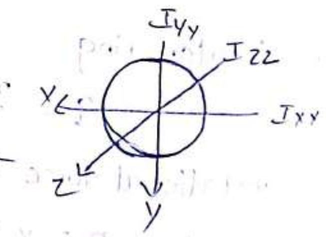
$G = \frac{\tau}{\frac{R\theta}{L}} \Rightarrow \frac{\tau \times L}{R\theta}$

$\frac{G\theta}{L} = \frac{\tau}{R} \longrightarrow$ (I)

Polar Moment of Inertia :- It is defined as the Moment of Inertia of the area about an axis which is perpendicular to the plane of the figure and passing through the center of gravity of the area.

It is denoted by J.

$J = \int_0^R r^2 \cdot dA$



For solid circular section :-

$J = I_{xx} + I_{yy}$

$= \frac{\pi D^4}{64} + \frac{\pi D^4}{64}$

$J = \frac{\pi D^4}{32}$

For Hollow circular section

$J = I_{xx} + I_{yy}$

$J = \frac{\pi}{64}(D^4 - d^4) \times 2$

$J = \frac{\pi}{32}(D^4 - d^4)$

To derive second part of the equation we have two methods

<u>Method-I</u>

$$\frac{\tau}{R} = \frac{G\theta}{L} \text{ ——— I}$$

considering $\frac{G\theta}{L}$ as constant therefore

$\frac{\tau}{R}$ = constant $\Rightarrow \tau \propto R$

Let us consider a circular Ring at a distance 'r' from the centre of centr shaft.

Let the shear stress be 'q' in the ring.

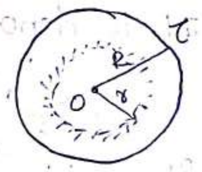
Since $\frac{\tau}{R}$ = constant,

$\therefore \frac{\tau}{R} = \frac{q}{r} \longrightarrow ①$

Shear stress = $\frac{\text{shear force (or) Rotational}}{\text{Area}}$

for circular ring

$q = \frac{\text{rotational force}}{dA}$



rotational force = $q \times dA$

dT = R.F $\times$ ⊥lr distance

$dT = (q \times dA) \times r$

from eq. ①, $q = \frac{\tau}{R} \times r$

$dT = \left(\frac{\tau}{R} \times r\right) dA \times r$

$dT = \frac{\tau}{R} \times r^2 \times dA$

I.O.B.S.

$T = \int_0^R \frac{\tau}{R} \times r^2 \times dA = \frac{\tau}{R}\int_0^R r^2 \cdot dA$

$T = \frac{\tau}{R} \times J$

$\frac{T}{J} = \frac{\tau}{R} \longrightarrow \text{II}$

from I & II

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

<u>Maximum</u> <u>Torque</u> <u>transmitted</u> <u>by</u> <u>Solid</u> <u>circular</u> <u>Sha</u>

(Method II)

$T = \int_0^R \frac{\tau}{R} \times r^2 \cdot dA \qquad [dA = 2\pi r \times dr]$

$= \int_0^R \frac{\tau}{R} \times r^2 (2\pi r \cdot dr)$

$= \frac{\tau}{R} \times 2\pi \int_0^R r^3 \cdot dr$

$= \frac{\tau}{R} \times 2\pi \left[\frac{r^4}{4}\right]_0^R$

$= \frac{\tau}{R} \times 2\pi \left[\frac{R^4}{4} - 0\right]$

$= \frac{\tau}{\cancel{R}} \times \cancel{2}\pi \times \frac{R^{\cancel{4}3}}{\cancel{4}_2}$

$= \tau \times \frac{\pi}{2} \times R^3$

W.K.T, $D = 2R \Rightarrow R = \frac{D}{2}$

$T = \tau \times \frac{\pi}{2} \times \left(\frac{D}{2}\right)^3$

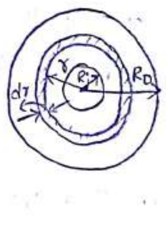
$$T = \tau \times \frac{\pi}{2} \times \frac{D^3}{8}$$

$$= \frac{\tau}{16} \times D^3 \times \pi$$

$$\boxed{\tau = \frac{16T}{\pi D^3}}$$

$$T = \frac{\pi D^3}{16} \times \tau$$

Max'm Torque Transmitted by hollow circular shaft



$$T = \int_{R_i}^{R_o} \frac{\tau}{R} \times r^2 \cdot dA$$

$$T = \frac{\tau}{R} \int_{R_i}^{R_o} r^2 \cdot (2\pi r)\, dr$$

$$= \frac{\tau}{R} \int_{R_i}^{R_o} 2\pi r^3 \cdot dr$$

$$= \frac{\tau}{R} \times 2\pi \int_{R_i}^{R_o} r^3 \cdot dr$$

$$= \frac{\tau}{R} \times 2\pi \times \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R} \times 2\pi \left[\frac{R_o^4}{4} - \frac{R_i^4}{4}\right]$$

$R_o = \frac{D_o}{2}$, $R_i = \frac{D_i}{2}$, $R = \frac{D_o}{2}$

$$= \frac{\tau}{\left(\frac{D_o}{2}\right)} \times \frac{2\pi}{4} \cdot \left[\frac{D_o^4}{16} - \frac{D_i^4}{16}\right]$$

$$= \frac{4\pi}{4} \times \frac{\tau}{D_o} \left[\frac{D_o^4 - D_i^4}{16}\right]$$

$$\boxed{T = \tau \times \left[D_o^4 - D_i^4\right] \times \frac{\pi}{16 D_o}}$$

$$\tau = \frac{16 T D_o}{\pi \left[D_o^4 - D_i^4\right]}$$

Strength of the Shaft (or) Torsional Rigidity

Flexural Rigidity = $E \times I$

Torsional Rigidity = $G \times J$

Strength of the shaft is defined as the maximum torque transmitted through the shaft.

$\therefore$ Torsional Rigidity = $G \times J$ = N-mm^2

where,

G = Modulus of Rigidity. N/mm^2

J = polar moment of inertia mm^4

Power transmitted through a shaft

Power Transmitted through a shaft $P = T \times \omega$ (Torque; $\omega \rightarrow$ Angular velocity)

$$= T \times \frac{2\pi N}{60} = \frac{2\pi N T}{60}$$

units:

$P_{W} \longrightarrow T^{N\text{-}m}$

$P_{kW} \longrightarrow T^{kN\text{-}m}$

Q) In a Hollow circular shaft of outer and Inner diameter 20 cm and 10 cm respectively; the shear stress not to exceed 40 N/mm^2. Find the maximum Torque which a shaft can safely Transmitted

Given D_o = 20 cm = 200 mm

Inner Diameter D_i = 10 cm = 100 mm

τ_{max} = 40 N/mm^2

T_{max} = ?

W.K.T Hollow circular shaft

$$\tau_{max} = \frac{16 T D_o}{\pi (D_o^4 - D_i^4)}$$

$$T = \frac{40 \times \pi \times (200^4 - 100^4)}{\pi (D_o^4 - D_i^4)}$$

$T = 58.90$ kN.m

Q) The shearing stress of a solid shaft is not to exceed 40 N/mm². when the torque transmitted is 20,000 N.m. Determine the diameter of the shaft.

Given $\tau_{max} = 40$ N/mm² $\left[T = \frac{\pi D^3}{16} \times \tau\right]$

$T = 20,000$ N.m

$$T = \frac{\pi \times D^3}{16} \times 40$$

$$D^3 = \frac{20,000 \times 10^3 \times 16}{\pi \times 40} = 136.55$$

Q) Find the maximum shear stress induced in a solid circular shaft of diameter 15 cm when shaft transmits 150 kw power at 180 rpm.

Given diameter = 15 cm

$P = 150$ kW $\left[P = T \times \frac{2\pi N}{60}\right]$

$N = 180$ rpm

$$P = \frac{2\pi N T}{60}$$

$$150 = \frac{2 \times \pi \times 180}{60} = \frac{150 \times 60}{2\pi \times 180} = 7.95 \text{ kN.m}$$

$$T = \frac{\pi D^3}{16} \times \tau$$

$$\tau = \frac{16 \times 7.95 \times 10^3 \times 10^3}{\pi \times 150^3} \Rightarrow \tau = 11.996 \text{ N/mm}^2$$

Q) A Hollow shaft is to transmit 300 k.w power at 80 rpm. If the shear stress is not to exceed 60 N/mm² and the internal diameter is 0.6 of the external diameter, find the external & internal diameter assuming the maximum Torque is 4 times the mean Torque.

Given $\tau_{max} = 60$ N/mm²

$P = 300$ k.W

$N = 80$ RPM

$d = 0.6\,D$

$T_{max} = 1.4\, T_{mean}$ $D = ?$ $d = ?$

W.k.T

$$P = \frac{2\pi N T_{mean}}{60}$$

$$\Rightarrow 300 \times 10^3 = \frac{2\pi \times 80 \times T_{mean}}{60}$$

$$T_{mean} = \frac{300 \times 10^3 \times 60}{2\pi \times 80}$$

$T_{mean} = 35.80 \times 10^3$ N.m

$T_{max} = 1.4\, T_{mean}$

$T_{max} = 1.4 \times 35.80 \times 10^3$

$= 50.12 \times 10^3$ N.m

$= 50.12 \times 10^6$ N.m

$$\tau_{max} = \frac{16\, T_{max}\, D}{\pi \times [D^4 - d^4]}$$

$$60 = \frac{16 \times 50.12 \times 10^6 \times D}{\pi \times [D^4 (0.6D)^4]}$$

$$[D^4 - d^4] = \frac{16\, T_{max}\, D}{\pi}$$

$$[D^4 - d^4] = \frac{16 \times 50.12 \times 10^6 \times D}{\pi \times 60}$$

$$D^4 - (0.6D)^4 = 4.254 \times 10^6 \times D$$

$$D^4 - 0.1296\,D^4 = 4.254 \times 10^6\, D$$

$$0.8704\, D^{\cancel{4}} = 4.254 \times 10^6\, \cancel{D}$$

$$D = 169.708$$

$$d = 0.6 \times D = 101.82 \text{ mm}$$

Q) A solid steel shaft as to transmit 75 kW at 200 rpm. Taking allowable shear stress 70 N/mm². Find suitable diameter for the shaft, if maximum Torque is transmitted at each revolution exceeds the mean by 30%.

Given
$P = 75$ kW
$N = 200$ rpm
$\tau = 70\ N/mm^2$
$T_{max} = 30\%$

W.K.T

$$P = \frac{2\pi N T}{60}$$

$$T_{max} = \frac{75 \times 10^3 \times 60}{2\pi \times 200}$$

$$T_{mean} = 3.58 \times 10^3\ N{\cdot}m$$

$$T_{max} = 1.3 \times T_{mean}$$

$$= 1.3 \times 3.58 \times 10^3$$

$$= 4.650 \times 10^3\ N{\cdot}m$$

$$\boxed{T = \frac{\pi D^3}{16} \times \tau}$$

$$D^3 = \frac{16\, T_{max}}{\pi \tau}$$

$$= \frac{16 \times 4.654 \times 10^3}{\pi \times 70}$$

$$= 3.386 \times 10^3$$

$$D = 69.70 \text{ mm}$$

Q) Determine the diameter of solid shaft which will transmit 300 kW at 250 rpm. The maximum shear stress should not exceed 30 N/mm² and twis should not be more than 1 degree in a shaft b of length of 2m. Take Modulus of rigidity = 1×1 N/mm²

Sol'
$P = 300$ kW
$N = 250$ rpm
$T_{max} = 30\ N/mm^2$
$\theta = 1°$
Shaft length $L = 2m$
$G = 1 \times 10^5\ N/mm^2$

W.K.T

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{300 \times 10^3 \times 60}{2\pi \times 250}$$

$$= 11.459 \times 10^3\ N{\cdot}m$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$J \rightarrow \frac{\pi d^4}{32}\ ; \quad R = \frac{d}{2}$$

$$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{d/2}$$

$$T = \frac{\pi d^3 \tau_{max}}{16}$$

$$d^3 = \frac{16T}{\pi \tau_{max}}$$

$$d^3 = \frac{16 \times 11.459 \times 10^3}{\pi \times 30} = 1.9453 \times 10^3$$

$$D = 124.8 \text{ mm}$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$J = \frac{\pi D^4}{32}$$

$$J = \frac{Tl}{G\theta}$$

$$\frac{\pi D^4}{32} = \frac{11.459 \times 10^6 \times 2000}{1 \times 10^5 \times 0.0175}$$

$$D^4 = \frac{11.459 \times 10^6 \times 2000 \times 32}{1 \times 10^5 \times 0.0175 \times \pi}$$

$$D = 107.469 \text{ mm}$$

<u>consider</u>

$$D = 124.8 \text{ mm}$$

$$T = \frac{\pi D^3}{16} \times \tau$$

$$\tau = \frac{16T}{\pi D^3} = \frac{16 \times 11.459 \times 10^6}{\pi \times 124.8^3}$$

$$\tau = 30.024 \text{ N/mm}^2$$

Note :- The shaft need to be design in such a way that it should be able to resist (or) withstand maximum torque and maximum shear stress.

Q) Determine the Diameter of a solid steel shaft which will transmit 90 KW at 160 rpm. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The maximum shear stress is limited to 60 N/mm². Take the value of modulus of rigidity 8×10^4 N/mm²

$P = 90$ KW. $N = 160$ rpm $\theta = 1° = 0.0175$ radian

$\tau_{max} = 60$ N/mm². $G = 8 \times 10^4$ N/mm²

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60 \times 90 \times 10^3}{2\pi \times 160} = 5.731 \times 10^3 \text{ N·m}$$

$$T = 5.371 \times 10^6 \text{ N·mm}$$

$$T = \frac{\pi D^3}{16} \times \tau$$

$$\frac{5.371 \times 10^6 \times 16}{\pi \times 60} = D^3$$

$$D^3 = 2.849 \times 10^{46}$$

$$D = 76.964 \text{ mm}$$

$$J = \frac{\pi D^4}{32} = 3.444 \times 10^6$$

w.k.T

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

$$\frac{5.371 \times 10^6}{3.444 \times 10^6} = \frac{8 \times 10^4 \times 0.0175}{l}$$

$$l = 897.709 \text{ mm}$$

Q. Two shafts of same materials and of same length are subjected to same torque. If the first shaft is solid circular section and second shaft is hollow circular section whose internal diameter is $2/3$ of the outside diameter and maximum stress developed in each shaft is same. Compare the weights of the shaft.

$$\frac{W_S}{W_H} = ? \qquad W = \text{Vol} \times \rho = (A \times l) \times \rho$$

$$\frac{W_S}{W_H} = \frac{\rho_S \times V_S}{\rho_H \times V_H} = \frac{\rho_S \times A_S \times L_S}{\rho_H \times A_H \times L_H}$$

$$\frac{A_S}{A_H} \Rightarrow \frac{\frac{\pi D^2}{4}}{\frac{\pi}{4}\left[D_0^2 - D_i^2\right]} \qquad \because D_i = \frac{2}{3} D_0$$

$$= \frac{\frac{\pi D^2}{4}}{\frac{\pi}{4} \times \left[D_0^2 - \frac{2}{3} D_0^2\right]} = \left[\frac{\frac{\pi D^2}{4}}{0.436 D_0^2}\right]$$

$$\frac{A_S}{A_H} = \frac{\frac{\pi D^2}{4}}{\frac{5\pi}{36} D_0^2} = \frac{9D^2}{5D_0^2} \quad \text{—— (1)}$$

Same torque & same shear stress:

$$\tau_{solid} = \tau_{Holl}$$

$$T_{sol} = T_{Hol}$$

$$\frac{16 T}{\pi D^3} = \frac{16 T D_0}{\pi\left[D_0^4 - D_i^4\right]} \qquad \left[D_i = \frac{2}{3} D_0\right]$$

$$\frac{16 T}{\pi D^3} = \frac{16 T D_0}{\pi\left[D_0^4 - \left(\frac{2}{3} \cdot D_0\right)^4\right]}$$

$$D_0^4 - \left(\frac{2}{3} D_0\right)^4 = D_0 \cdot D^3$$

$$D_0^4 - \frac{16}{81} D_0^4 = D_0 \cdot D^3$$

$$0.802\, D_0^{\cancel{4}\,3} = \cancel{D_0} \cdot D^3$$

$$0.803\, D_0^3 = D^3$$

$$(0.803)^{1/3} D_0 = D.$$

$$\boxed{0.929\, D_0 = D.}$$

in (1)

$$\frac{A_S}{A_H} = \frac{9}{5} \cdot \frac{D^2}{D_0^2} \Rightarrow \frac{9}{5}\left[\frac{0.93 D_0}{D_0}\right]^2 = 1.55$$

$$\frac{W_S}{W_H} = \frac{\rho_S \times V_S}{\rho_H \times V_H} = \frac{\cancel{\rho_S} \times A_S \times \cancel{L_S}}{\cancel{\rho_H} \times A_H \times \cancel{L_H}} = \frac{A_S}{A_H} = 1.55$$

Combined bending and Torsion:

When a shaft is transmitting torque (or) power it is subjected to shear stress at the same time shaft is also subjected to Bending moment due to gravity or inertia force.

Due to: Bending moment, bending stresses also develop in the shaft, hence each particle in a shaft is subjected to shear stress & Bending stress.

Consider any point in the ...

Let T = Torque

D = Diameter

M = Bending moment of the section

τ = shear stress at that point produced by Torque 'T'

$f = \sigma_b$ = Bending stress due to bending moment

By using Bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{M \times \cancel{64}^{32}}{\pi D^{\cancel{4}3}} \times \frac{\cancel{D}}{\cancel{2}}$$

$$\boxed{\sigma_b = \frac{32 M}{\pi D^3}}$$

Torsion equation

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

$$\tau = \frac{T}{J} \times R$$

$$= \frac{T \times \cancel{32}^{16}}{\pi D^{\cancel{4}3}} \times \frac{\cancel{D}}{\cancel{2}}$$

$$\boxed{\tau = \frac{16 T}{\pi D^3}}$$

Principal stresses

$$\sigma_{1,2} = \left[\frac{\sigma_x + \sigma_y}{2}\right] \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left[\frac{\sigma_b}{2}\right] \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{32}^{16} M}{\pi D^3} \pm \sqrt{\left(\frac{\cancel{32}^{16} M}{\cancel{2}\pi D^3}\right)^2 + \left(\frac{16 T}{\pi D^3}\right)^2}$$

$$= \frac{16 M}{\pi D^3} \pm \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

$$\boxed{\sigma_{1,2} = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]} \longrightarrow \text{For solid Circular Shaft}$$

... for Hollow circular shaft

$$\sigma_{1,2} = \frac{16 D_o}{\pi\left[D_o^4 - D_i^4\right]}\left[M \pm \sqrt{M^2 + T^2}\right]$$

→ The angle made by the plane of maximum shear with the Normal cross section is given by

$$\tan 2\theta = \frac{2\tau}{\sigma_b}$$

Q) A solid shaft of diameter 80 mm is subjected to a twisting moment of 8 MN-mm. and a Bending moment of 5 MN-mm at a point. Determine i) principal stresses and ii) position of plane on which they act.

A:- $D = 80$ mm , $T = 8$ MN·mm $= 8 \times 10^6$ N·mm

$M = 5$ MN-mm $= 5 \times 10^6$ N·mm

$$\sigma_1 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi \times 80^3}\left[5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right]$$

$= 143.577$ N/mm²

$$\sigma_2 = \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$$

$= -44.10$ N/mm²

$$\tan 2\theta = \frac{2\tau}{\sigma_b}$$

$$\tau = \frac{16 T}{\pi D^3} = \frac{16 \times 8 \times 10^6}{\pi \times 80^3} = 79.57 \text{ N/mm}^2$$

$$\sigma_b = \frac{32 M}{\pi D^3}$$

$= 99.47 \ N/mm^2$

$$Tan 2\theta = \frac{2\tau}{\sigma_b} = \frac{2 \times 79.57}{99.47}$$

$2\theta = 57.99 \Rightarrow \theta = 28.99$

$\theta = 28^\circ 59' 42''$

Tan $90 + \theta \Rightarrow 90 + 28.99$

$= 118.99 = 118^\circ 59' 24''$

II-part

Springs

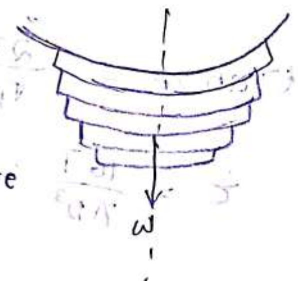
Springs are the elastic members which distract and regain their original shape after load removal. They are used to absorb the shock or impact loadings in railway wagons, coaches, vehicles

Types of Springs.

1) Laminated (or) Leaf spring
2) Helical springs.

1) Laminated Spring

These springs are used to wagons, etc.



→ Spring consist of plates which are of uniform width and thickness and are of different length but having same angle.

→ each plate is free to slide relative to other Spring

2) Helical Springs :-

Helical springs are the thick springs in which wires are coiled into helics. Those are two types :-

(i) closed coil helical springs.

* Helix angle (α) is very small ($\alpha \leq 10^\circ$)
* As the helix angle is very small. Therefore Bending effect is ignored.
* we assume the coils of closed coil springs withstand pure torsion.

ii) open coil helical springs.

* Helix angle is greater than 10°
* As the Helix angle is large compared to closed coil springs. Therefore it can resist both bending moment and torsion.

* closed coil helical Spring subjected to axial load. (or) Axial pull.

Let 'd' be the diameter of the spring

'p' be the pitch

n = number of coils

R = Radius of the coil (or) Mean radius of spring wire

W = axial load (or) axial pull.

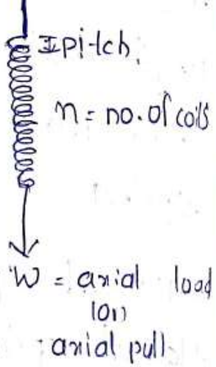
G = Modulus of Rigidity

θ = angle of twist

τ = Maximum shear stress

δ = Deflection of the Spring

l = length of the wire.



(1) Expression for Maximum shear stress

Generated Torque = T $\Rightarrow \tau = \frac{16T}{\pi d^3} \Rightarrow T = \frac{\pi d^3}{16} \times \tau$

External Twisting moment = $W \times R$

For equilibrium, Twisting moment = Torque

$$W \times R = \frac{\pi d^3}{16} \times \tau$$

$$\boxed{\tau = \frac{16WR}{\pi d^3}}$$

2) Expression for deflection

W.K.T strain energy stored in a Spring is = work done

$$U = \frac{\tau^2}{4G} \times Vol$$

$$\tau = \frac{16WR}{\pi d^3}$$

$$U = \frac{1}{4G}\left[\frac{16WR}{\pi d^3}\right]^2 \times vol.$$

$$= \frac{1}{4G}\left[\frac{16WR}{\pi d^3}\right]^2 \times Area \times length$$

length of one coil = $2\pi R$

length of n no. of coils = $2n\pi R$

Area = $\frac{\pi d^2}{4}$

$$U = \frac{1}{4G} \times \frac{\cancel{256}^{32} W^2 R^2}{\pi d^3 \times \pi d^3} \times \frac{\pi d^2}{\cancel{4}} \times 2n\pi R.$$

$$\boxed{U = \frac{32 W^2 R^3 n}{G d^4}}$$

$$U = \frac{1}{2} \times W \times \delta$$

$$\delta = \frac{2U}{W} = \frac{64 W R^3 n}{G d^4}$$

$$\boxed{\delta = \frac{64 W R^3 n}{G d^4}}$$

iii) Expression for Spring constant (k) | stiffness factor

It is denoted by k.

It is defined as ratio of load to the deflection

Mathematically, $k = \frac{W}{\delta}$

$$k = \frac{W}{\frac{64WR^3n}{Gd^4}} \Rightarrow \frac{W G d^4}{64 W R^3 n} = \boxed{\frac{Gd^4}{64R^3n} = k}$$

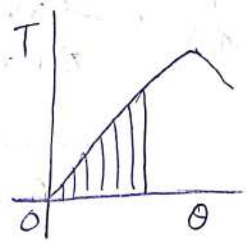
iv) Expression for proof load (w)

It is that maximum load that a spring can carry without undergoing any deformation.

$$\tau_{max} = \frac{16 W_{max} R}{\pi d^3} \Rightarrow W_{max} = \frac{\tau_{max} \times \pi d^3}{16R}$$

Expression for strain energy of a shaft Subjected to torsion.

S.E stored = work done = area under load curve in graph



$= \frac{1}{2} \times \theta \times T$

$= \frac{1}{2} \times T \times \theta$ $\qquad \frac{T}{J} = \frac{G\theta}{l}$

$= \frac{1}{2} \times T \times \frac{Tl}{GJ}$ $\qquad \theta = \frac{Tl}{GJ}$

$= \frac{T^2 L}{2GJ} \Rightarrow \frac{T^2 l}{2GJ}$ $\qquad \tau = \frac{16T}{\pi d^3}$

$= \frac{l}{2G} \times \frac{T^2}{J}$

$= \frac{L}{2G} \times \frac{\pi d^3}{16} \times \tau \times \frac{\pi d^3}{16} \times \tau \times \frac{1}{J}$

$= \frac{L}{2G} \times \tau^2 \times \frac{\pi d^3}{16} \times \frac{\pi d^3}{16} \times \frac{32}{\pi d^4}$

$= \left\{ \frac{L \times \tau^2 \times \pi d^2}{16G} \right\}$

$= \frac{l}{2G} \times \tau^2 \times \frac{\pi d^2}{8}$

$= \frac{\tau^2}{16G} \times \pi d^2 \times l$

$= \frac{\tau^2}{4G} \times \underbrace{\left(\frac{\pi d^2}{4}\right)}_{Area} \times l$

$\boxed{U = \frac{\tau^2}{4G} \times vol}$

Q) A railway wagon weighing 40 kN and moving with a Speed of 8 km/h is supported by a Buffer of 4 springs whose maximum allowable compression is 150 mm. Find out the number of turns in each spring, if the dia of the spring wire is 14 mm and the dia of the coil is 80 mm. Assume Modulus of rigidity is 84 GN/m²

(Load = 40 kN Speed = 8 km/h.)

$W_R = 40$ kN, Speed (v) = 8 kmph $= 8 \times \frac{5}{18}$

$= 2.22$ m/s

no. of springs = 4

Max'm deflection (δ) 150 mm

d = 14 mm, D = 80 mm $\quad$ R = 40 mm

$G = 84 \times 10^3$ N/mm²

W.k.t

SE absorbed = workdone

$\frac{1}{2} \times W \times \delta = K.E$

$\Downarrow$

$\frac{1}{2} m v^2$

$W = mg$

$m = \frac{W_R}{g}$

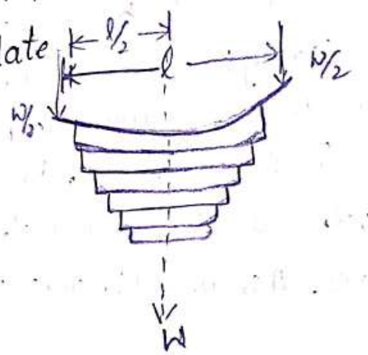
$\frac{1}{2} \times \frac{W_R}{g} \times v^2 = \frac{1}{2} \times W \times \delta$

$\Downarrow$

$\frac{1}{2} \times \frac{40 \times 10^3}{9.81} \times (2.22)^2$

$= 10.047 \times 10^3$

Expression for Maximum Bending stress developed in the plate



Max. BM @ centre $= \frac{W}{2} \times l/2 \Rightarrow \frac{Wl}{4}$

M.I $= \frac{bt^3}{12}$

$\frac{M}{I} = \frac{f}{y}$

$\frac{12M}{bt^3 2} = \frac{\sigma_b}{t} \times 2$ —— ①

$\frac{6 \times Wl}{4bt^2} = \sigma_b$

$\sigma_b = \frac{3Wln}{2bt^2}$ → ②

$M = \frac{\sigma_b \cdot bt^2}{6}$

M shared by 'n' no of plates

$M = \frac{\sigma_b \cdot bt^2 n}{6}$ → I

Max'm BM = resisting Moment : $\frac{Wl}{4} = \frac{\sigma_b bt^2 n}{6}$

$\frac{6 \times Wl}{4bt^2} = \sigma_b$

$$\boxed{\sigma_b = \frac{3Wl}{2bt^2 n}} \rightarrow ②$$

II. Columns and struts

Columns :-

Columns are the vertical members always subjected to compressive load (vertical load or Axial load)

i) Short column: A column that fails essentially by direct crushing at ultimate load.

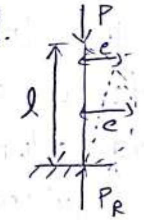
$P_c \rightarrow$ crushing load $\sigma_c \rightarrow$ crushing stress (direct stress)

$\sigma_c = \frac{P_c}{A} \Rightarrow P_c = \sigma_c \times A$

ii) Long column :- columns which are considerabilily long in compaiision to their lateral dimensions, and essentially failed by buckling or cripling load or cripling load.

$P_{cr} \rightarrow$ Buckling (or) cripling load.

$\sigma_b \rightarrow$ Bending stress $\Rightarrow \frac{P_e}{Z}$



$\frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \sigma_b = \frac{M}{I} \times y$

$= P_e \times \frac{y}{I}$

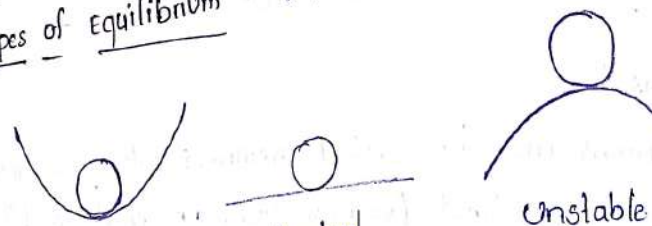
$= \frac{P_e}{Z}$

iii) Intermediate or Medium column :- A column that fails by both direct crushing and bending stress then it is known as intermediate or Medium colum

$\sigma_{fail} = \sigma_c + \sigma_b$

$= \frac{P_c}{A} + \frac{P_e}{Z}$

Types of Equilibrium



Stable (i) Neutral (ii) Unstable (iii)

i) Stable Equilibrium :-

If a small axial load is applied to a column it gets deform and returns to its orginal position after the removal of the load. is said to be in stable equilibrium.

ii) Neutral Equilibrium

If the load applied is equal to ultimate load for a short column and buckling load for a long column then the column is said to be in neutral Equilibrium.

iii) Unstable equilibrium

If a small load is applied to a column it can't regain it's original position. then the column is said to be in unstable equilibrium.

Buckling load (or) crippling load (or) critical load

The load at which the column starts buckling laterally is called as buckling/cripling/critical load.

→ It is denoted by P_{cr}

Note :- To find the buckling load the formula used for different end conditions of a column is

Eulers formula.

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

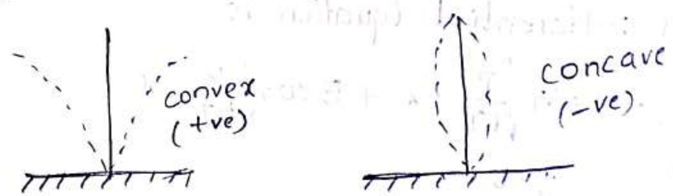
Assumptions in Euler's theory

1) The column is initially straight and the load applied is axially (axial)
2) The cross - section of the column is uniform throughout the length.
3) The column material is perfectly elastic, homogene and isotrophic and also obeys the hook's law
4) the length of the column is very large compare to lateral dimensions.
5) The direct stress is very small compared to the bending stress.
6) The column fails by buckling
7) The self weight of column is negligible

End conditions in long columns :-

i) Both ends are hinged
ii) one end is fixed & other end is free
iii) Both ends are fixed
iv) One end is fixed & other end is hinged

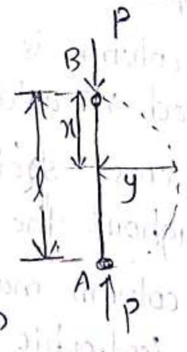
Sign conventions :-



i) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$ | ii) $\frac{d^2y}{dx^2} + \alpha^2 y = a$

$y = A\sin\alpha x + B\cos\alpha x$ | $y = A\sin\alpha x + B\cos\alpha x + a$

1) Both ends are hinged



consider a long column. AB which is hinged at both ends.

Take a section at a distance 'x' from B which is having a deflection of 'y'

Let 'L' be the length of the column. Therefore Bending moment at the section $= -P \times y$

$$\therefore \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$M = -Py$$

$$\frac{d^2y}{dx^2} = \frac{-P}{Ey} \cdot y$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$$

Compare with standard Differential Equation

$$\frac{d^2y}{dx^2} + \alpha^2 y = 0$$

$$\alpha = \sqrt{\frac{P}{EI}}$$

Sol'n of Differential Equation is.

$$y = A\sin\sqrt{\frac{P}{EI}} \cdot x + B\cos\sqrt{\frac{P}{EI}} \cdot x$$

i) @ $x = 0$, $y = 0$

ii) @ $x = l$, $y = 0$

to find A & B:

i) At $x = 0$, $y = 0$

$$y = A\sin\sqrt{\frac{P}{EI}} \cdot x + B\cos\sqrt{\frac{P}{EI}} \cdot x$$

$$0 = A(0) + B(1)$$

$$B = 0$$

ii) At $x = l$, $y = 0$

$$y = A\sin\sqrt{\frac{P}{EI}} \cdot x$$

$$0 = A\sin\left(\sqrt{\frac{P}{EI}} \cdot l\right) \qquad [\because A \neq 0]$$

$$\sin\left(\sqrt{\frac{P}{EI}} \cdot l\right) = 0.$$

Note :-

1) Since the column is having two boundary conditions at B

i) at $x = 0$, $y = 0$ and at $x = l$, $y = 0$

2) $A \neq 0$ since, otherwise all the points will have zero deflection

$$\sqrt{\frac{P}{EI}} \cdot l = n\pi$$

S.O.B.S

$$\frac{P_{cr}}{E.I.} \cdot l^2 = n^2\pi^2$$

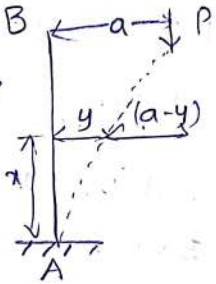
$$P_{cr} = \frac{n^2\pi^2 EI}{l^2}$$

Since the crippling is minimum we should take n=

$$\therefore P_{cr} = \frac{\pi^2 EI}{l^2}$$

2) One end is fixed & other ...

Consider a long column AB which is fixed at one end and other is free. Take a section 'x' at a distance of 'x' from A is having deflection y



$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$M = +P(a-y)$$

$$\frac{d^2y}{dx^2} = \frac{P(a-y)}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{Pa}{EI} - \frac{Py}{EI} \Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 \cdot a$$

$$y = A\sin\sqrt{\frac{P}{EI}}\cdot x + B\cos\sqrt{\frac{P}{EI}}\cdot x + a$$

i) at $x = 0,\ y = 0$. ii) at $x=0,\ \frac{dy}{dx}=0$ iii) at $x=l,\ y=a$

i) at $x=0$, $y=0$.

$$0 = A\sin\sqrt{\frac{P}{EI}}\cdot(0) + B\cos\sqrt{\frac{P}{EI}}(0) + a$$

$$\left(B\sqrt{\frac{P}{EI}} = \right) \quad \boxed{B = -a}$$

ii) at $x=0$, $\frac{dy}{dx}=0$

$$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\cdot x\cdot\left(\sqrt{\frac{P}{EI}}\right) - B\sin\sqrt{\frac{P}{EI}}\cdot x\sqrt{\frac{P}{EI}} + 0$$

$(B = -a)$

$$0 = A(1)\sqrt{\frac{P}{EI}} - (-a)(0) + 0$$

$$A\sqrt{\frac{P}{EI}} = 0$$

$$\because \sqrt{\frac{P}{EI}} \neq 0 \ ; \ \therefore A = 0$$

$$y = -a\cos\sqrt{\frac{P}{EI}}\cdot x + a$$

iii) At $x = l$, $y = a$

$$a = -a\cos\sqrt{\frac{P}{EI}}\cdot l + a$$

$$a = a\left[-\cos\sqrt{\frac{P}{EI}}\cdot l + 1\right]$$

$$\cancel{1} = -\cos\sqrt{\frac{P}{EI}}\cdot l + \cancel{1}$$

$$\cos\sqrt{\frac{P}{EI}}\cdot l = 0$$

$$\sqrt{\frac{P}{EI}}\cdot l = \frac{n\pi}{2}$$

S.O.B.S

$$\frac{P_{cr}}{EI}\cdot l^2 = \frac{n^2\pi^2}{4}$$

$$P_{cr} = \frac{n^2\pi^2 EI}{4l^2}$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{4l^2}}$$

3) Both ends are fixed

$$M = -Py + M_0$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{-Py + M_0}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI} + \frac{M_0}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI} \times \frac{P}{P}$$ Multiply & divide by P to convert to 2nd eqn

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{P} \times \left(\frac{P}{EI}\right)$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 \cdot q$$

$$y = A \sin\sqrt{\frac{P}{EI}}\cdot x + B\cos\sqrt{\frac{P}{EI}}\cdot x + \frac{M_0}{P}$$

1) @ $x = 0$, $y = 0$

2) @ $x = 0$, $\frac{dy}{dx} = 0$

3) @ $x = l = y = 0$

1) @ $x = 0$, $y = 0$

$$0 = A\sin\sqrt{\frac{P}{EI}}(0) + \underbrace{B\cos\sqrt{\frac{P}{EI}}(0)}_{(1)} + \frac{M_0}{P}$$

$$B = \frac{-M_0}{P}$$

2) @ $x = 0$, $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\underbrace{A\sin}_{\cos x}\sqrt{\frac{P}{EI}}(0) + \underbrace{B\cos}_{\sin x}\sqrt{\frac{P}{EI}}(0) + \frac{M_0}{P}$$

$$0 = A(0) + \left(\frac{-M_0}{P}\right) + \frac{M_0}{P}$$

$$0 = -\frac{M_0}{P} + \frac{M_0}{P}$$

$$A = 0$$

$$0 = A(1)\left(\sqrt{\frac{P}{EI}}\right) - 0$$

$$\because \sqrt{\frac{P}{EI}} \neq 0$$

$$\therefore A = 0$$

$$0 = A\sin\sqrt{\frac{P}{EI}}(l) + B\cos\sqrt{\frac{P}{EI}}(l) + \frac{M_0}{P}$$

$$0 = 0 + \frac{-M_0}{P}\cos\sqrt{\frac{P}{EI}}(l) + \frac{M_0}{P}$$

$$-\frac{M_0}{P}\cos\sqrt{\frac{P}{EI}}\,l + \frac{M_0}{P} = 0$$ $\frac{M_0}{P}\left(-1\cos\sqrt{\frac{P}{EI}} + 1\right.$

$$\frac{M_0}{P}\left[1 - \cos\sqrt{\frac{P}{EI}}\,l\right] = 0$$

$$\because \frac{M_0}{P} \neq 0$$

$$\therefore -\cos\sqrt{\frac{P}{EI}}\cdot l + 1 = 0$$

$$\cos\sqrt{\frac{P}{EI}}\cdot l = +1$$ $\cos(360)$ $\cos(4\pi) = 1$

$$\sqrt{\frac{P}{EI}}\cdot l = n\pi$$ $[n = 2, 4 \ldots]$ even

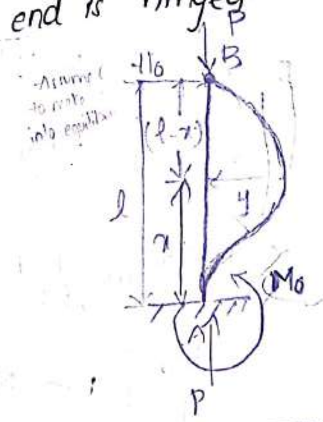
S.O.B.S

$$\frac{P}{EI}\cdot l^2 = n^2\pi^2$$

$$P_{cr} = \frac{n^2\pi^2 EI}{l^2}$$ [min value should be 'n'.

$$\boxed{P_{cr} = \frac{4\pi^2 EI}{l^2}}$$

iv) <u>One end is fixed other end is hinged</u>



$M = -Py + H_0(l-x)$

$\frac{d^2y}{dx^2} = \frac{M}{EI}$

$= \frac{-Py + H_0(l-x)}{EI}$

$\frac{d^2y}{dx^2} = \frac{-Py}{EI} + \frac{H_0(l-x)}{EI}$

Note

$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{H_0(l-x)}{EI}$

$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{H_0(l-x)}{P} \times \frac{P}{EI}$

$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \left[\frac{H_0(l-x)}{P}\right] \times \frac{P}{EI}$

$y = A\sin\sqrt{\frac{P}{EI}}\cdot x + B\cos\sqrt{\frac{P}{EI}}\cdot x + \frac{H_0(l-x)}{P}$

i) $x=0, y=0$ ii) $x=0, \frac{dy}{dx}=0$ iv) $x=l$ $y=0$

i) at $x=0, y=0$

$0 = A\sin\sqrt{\frac{P}{EI}}\cdot(0) + B\cos\sqrt{\frac{P}{EI}}(0) + \frac{H_0(l-x)}{P}$

$B = \frac{-H_0(l-(0))}{P}$

$B = \frac{-H_0 l}{P}$

ii) at $x=0$, $\frac{dy}{dx}=0$

$\frac{dy}{dx} = A\sin\sqrt{\frac{P}{EI}}(0) + B\cos\sqrt{\frac{P}{EI}}(0) + \frac{H_0(l-x)}{P}$

$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\cdot x\cdot\sqrt{\frac{P}{EI}} - B\sin\sqrt{\frac{P}{EI}}\cdot x\cdot\sqrt{\frac{P}{EI}} - \frac{H_0}{P}(1)$

$0 = A(1)\sqrt{\frac{P}{EI}} - \frac{H_0}{P}$

$A\sqrt{\frac{P}{EI}} = \frac{H_0}{P}$

$A = \frac{H_0}{P}\sqrt{\frac{EI}{P}}$

iii) $x=l$, $y=0$

$y = \frac{H_0}{P}\sqrt{\frac{EI}{P}}\cdot\sin\sqrt{\frac{P}{EI}}\cdot x - \frac{H_0 l}{P}\cos\sqrt{\frac{P}{EI}}\cdot x + \frac{H_0(l-x)}{P}$

$0 = \frac{H_0}{P}\sqrt{\frac{EI}{P}}\cdot\sin\sqrt{\frac{P}{EI}}\cdot l - \frac{H_0 l}{P}\cos\sqrt{\frac{P}{EI}}\cdot l + \cancel{\frac{H_0(l-l)}{P}}$

$0 = \frac{H_0}{P}\left[\sqrt{\frac{EI}{P}}\cdot\sin\sqrt{\frac{P}{EI}}\cdot l - l\cos\sqrt{\frac{P}{EI}}\cdot l\right]$

$\because \frac{H_0}{P} \neq 0$; $\sqrt{\frac{EI}{P}}\sin\sqrt{\frac{P}{EI}}\cdot l - l\cos\sqrt{\frac{P}{EI}}\cdot l = 0$

$\sqrt{\frac{EI}{P}}\cdot\sin\sqrt{\frac{P}{EI}}\cdot l - l\cos\sqrt{\frac{P}{EI}}\cdot l = 0$

$\sqrt{\frac{EI}{P}}\cdot\sin\sqrt{\frac{P}{EI}}\cdot l = l\cos\sqrt{\frac{P}{EI}}\cdot l$

$(\sqrt{\frac{EI}{P}}$ sin)

$\frac{\sin\sqrt{\frac{P}{EI}}\cdot l}{\cos\sqrt{\frac{P}{EI}}\cdot l} = l\cdot\sqrt{\frac{P}{EI}}$

$$\tan\left(\sqrt{\frac{P}{EI}}\cdot l\right) = \sqrt{\frac{P}{EI}}\cdot l$$

$\tan\theta \approx \theta$

$\theta = 4.5$ radians → standard valu

$$4.5 = \sqrt{\frac{P}{EI}}\cdot l$$

S.O.B.S

$$\frac{P_{cr}}{EI}\cdot l^2 = [20.25] \rightarrow 2\pi^2$$

$$\boxed{P_{cr} = \frac{2\pi^2 \cdot EI}{l^2}}$$

Effective length :

Effective length of a given column with a given end condition is the length of a equivalent column of a same material and cross section with hinge ends and having the value of crippling load equal to the given column.

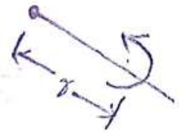
It is denoted by l_e

End conditions	crippling load	Effective length	Relation b/w effective length and Actual length
Both ends pinned	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = l$
One end is fixed and other is free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = 2l$
Both ends are fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = \frac{l}{2}$
(4) One end is fixed other end is hinged.	$\frac{2\pi^2 \cdot EI}{l^2}$	$\frac{\pi^2 EI}{l_e^2}$	$L_e = \frac{l}{\sqrt{2}}$

Radius of Gyration (r)

It is defined as the minimum distance from the centroid to the axis of rotation

It is denoted by 'r'



$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

I_{min} = moment of Inertia

A = Area of cross section

Slenderness ration (λ)

It is defined as the ratio of effective length of a column to the least radius of gyration

$$\lambda = \frac{l_e}{r_{min}}$$

Crippling stress :-

It is defined as ratio of cripling load to Area

$$P_{cr} = \frac{\pi^2 E I}{l^2} \qquad \because \sigma_{cr} = \frac{P_{cr}}{A}$$

$$\sigma_{cr} = \frac{\pi^2 E I}{l_e^2 \times A} \qquad r = \sqrt{\frac{I}{A}}$$

$$= \frac{\pi^2 E \times r^2}{l_e^2} \qquad r^2 = \frac{I}{A}$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{l_e^2}{r^2}\right)}$$

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}}.$$

Limitations of Euler's theory :

1) Euler's theory is only used for very long columns.

2) For short columns slenderness ratio 'λ' will be greater than that calculated with Euler's equation

Example ! crushing stress = 330 N/mm^2, $E = 2.1 \times 10^5\ N/$

$\lambda = ?$

Given $\lambda = 80$, verify λ calculated with euler's equation.

$$\lambda = \sqrt{\frac{\pi^2 E}{\sigma_{cr}}} = \sqrt{\frac{\pi^2 \times 2.1 \times 10^5}{330}} = 79.25.$$

$79.25 < 80$ $\qquad \lambda 7$

Rankine's Formula :

$$\frac{1}{P} = \frac{1}{P_C} + \frac{1}{P_E}$$

where : P = Rankines load.

P_C = crushing load.

P_E = Eulers crippling load.

for short columns :-

- for short column effective length l_e is small

w.k.t

$$P_E = \frac{\pi^2 E I}{l_e^2}$$

When l_e is small P_E becomes large.

$\therefore \frac{1}{P_E}$ becomes small.

$$\frac{1}{P_E} = \frac{1}{P_C} + \cancel{\left(\frac{1}{P_E}\right)} \rightarrow \text{negligible}$$

$\therefore \boxed{P = P_c}$

- for long columns.

For long columns, l_e is large

w.k.T $P_E = \frac{\pi^2 E I}{l_e^2}$

when l_e is large, P_E becomes small

$\therefore \frac{1}{P_E}$ becomes large.

$\frac{1}{P} = \frac{1}{P_E} + \left[\frac{1}{P_c}\right]$ negligible

$\therefore \boxed{P = P_E}$

$\Rightarrow \frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$

$\frac{1}{P} = \frac{P_c + P_E}{P_c \cdot P_E}$

$P = \frac{P_c \cdot P_E}{P_c + P_E}$

Divide with P_E.

$P = \frac{P_c}{\frac{P_c}{P_E} + \frac{P_E}{P_E}}$

$= \frac{P_c}{1 + \frac{P_c}{P_E}}$

$P_c = \sigma_c \times A$

$P_E = \frac{\pi^2 E I}{l_e^2} \Rightarrow \frac{(\sigma_c \times A)}{1 + \frac{\sigma_c \times A \cdot l_e^2}{\pi^2 E I}}$

$r = \sqrt{\frac{I}{A}} \Rightarrow I = Ar^2$

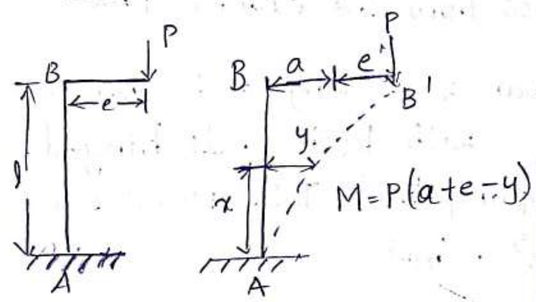
$= \frac{\sigma_c \times A}{1 + \frac{\sigma_c \times A \times l_e^2}{\pi^2 E (A r^2)}}$ w.k.T; $\lambda = \frac{l_e}{r}$

$= \frac{\sigma_c \times A}{1 + \frac{\sigma_c \cdot \lambda^2}{\pi^2 E}}$

$\boxed{P_R = \frac{\sigma_c \times A}{1 + a\lambda^2}}$ $\left[a = \text{rankine's constant} = \frac{\sigma_c}{\pi^2 E} \right]$

Columns with eccentric loading

i) Secant Formula :-



$\sigma_{max} = \sigma_o + \sigma_b$

$\sigma_{max} = \frac{P}{A} + \frac{P \times e \times \sec\left[\frac{l_e}{2}\right.}{Z}$

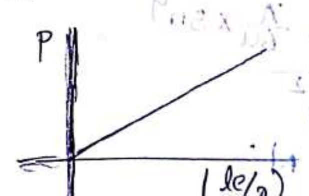
ii) straight line formula

$\frac{P}{A} = \sigma_c - n\left[\frac{l_e}{r}\right]$

where P = crippling load.

$\frac{l_e}{r}$ = slenderness ratio

n = constant depends upon the column



Note : The graph plotted with P against $\frac{l_e}{r}$ gives a straight line. So this formula is called straight line formula

ii) Perry's formula :-

$$\left(\frac{\sigma_{max}}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_{cr}}\right) = 1.2\left(e \cdot \frac{y_c}{r^2}\right)$$

where

$\sigma_{max} = \sigma_0 + \sigma_b$.

σ_0 = direct stress., σ_b = bending stress

σ_{cr} = Eulers crippling load.

e = eccentricity

r = radius of gyration

y_c = distance from the extreme fibre

Q) A solid round bar 3 m long and 5 cm in dia is used as a strut. with both ends hinged. Determine the crippling load. Take Modulus of Elasticity $E = 2 \times 10^5$ N/mm²

Given

$l = 3$ m $= 3000$ mm

dia $= 5$ cm $= 50$ mm

$E = 2 \times 10^5$ N/mm²

$P_{cr} = ?$ $[l_e = l]$

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

$$P_{cr} = \frac{\pi^2 \times 2 \times 10^5 \times \frac{\pi}{64} \times 50^4}{(3000)^2}$$

$P_{cr} = 67.26 \times 10^3$ N.

Q) A column of timber section 15 cm × 20 cm is 6 m long. Both the ends are being fixed. If the young's Modulus of timber = 17.5 KN/mm². Determine

i) Crippling load ii) Safe load of column if FOS is 3.

Given :

$b = 15$ cm ⇒ 150 mm

$d = 20$ cm ⇒ 200 mm.

$l = 6$ m ⇒ 6000 mm

Both ends are fixed

$E = 17.5 \times 10^3$ N/mm²

$I_{xx} = \frac{bd^3}{12} = 100 \times 10^6$

$I_{min\,yy} = \frac{db^3}{12} = 56.2 \times 10^6$ mm⁴

$$P_{cr} = \frac{4\pi^2 E I}{l_e^2}$$

$$= \frac{4 \times \pi^2 \times 17.5 \times 10^3 \times 56.2 \times 10^6}{(6000)^2}$$

$P_{cr} = 1.079 \times 10^6$ N.

$P_{cr} = 1.079 \times 10^3$ kN.

ii) Safe load $= \frac{P_{cr}}{FOS}$

$$= \frac{1079.4}{3} = 359.8 \text{ kN}$$

Q) A simply supported beam of length 4 m subjected to a U.DL of 30 kN/m over the whole span and deflect 15 mm at the centre. Determine the crippling load when this beam is used as a column with the following conditions

i) One end is fixed & other end ...

ii) Both ends are pin jointed .

Given

length : 4 m = 4000 mm

UDL = 30 kN/m = 30 N/mm

δ = 15 mm

$\delta = \frac{5 w l^4}{384 E I}$ $I =$

$EI = \frac{5 \times 30 \times (400)^4}{384 \delta}$ $N/mm^2 \times mm^4$

$EI = 6.66 \times 10^{12} N \cdot mm^2$ $N/m^2 \times mm$

i) $P_{cr} = \frac{2\pi^2 EI}{l^2}$

$P_{cr} = 8.2164 \times 10^6 N$

$P_{cr} = 8216 \times 10^3 kN$

ii) $P_{cr} = \frac{\pi^2 EI}{l^2}$

$= 4.106 \times 10^6 \Rightarrow 4106 \times 10^3 N$

Q) A hollow cylindrical cast iron is 4 m long with both ends fixed . Determine the minimum diameter of the column . If it has to carry a safe load of 250 kN with a F.O.S of 5.. Take Internal dia as 0.8 times the external dia. Take crushing stress = 550 N/mm^2 and Rankines constant $\frac{1}{1600}$ in Rankines formula

Given :-

length = 4 m = 4000 mm

$L_e = \frac{l}{2} = \frac{4000}{2} = 2000$ mm

Safe load = 250 kN = 250×10^3 N

F.O.S = 5

Internal dia = 0.8 D

crushing strength : $\sigma_{cr} = 550 N/mm^2$

Rankines constant $a = \frac{1}{1600}$

P_R = Safe load × FOS

= 250 × 5 = 1250 kN

$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[D^2 - (0.8D)^2\right] \Rightarrow 0.2$

$r = \sqrt{\frac{I}{A}} \Rightarrow I = \frac{\pi}{64}\left[D^4 - (0.8d)^4\right]$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}(D^4 - (0.8D)^4)}{\frac{\pi}{4}(D^2 - (0.8D^2))}}$

$r = \sqrt{\frac{\frac{1}{16}(0.59 D^4)}{0.36 D^2}} \Rightarrow \frac{1}{4}(1.27D)$

$r = 0.32D$

$\lambda = \frac{2000}{0.32D}$

$\lambda = \frac{6250}{D}$

$P_R = \frac{\sigma_c \times A}{1 + a\lambda^2}$

$1250 \times 10^3 = \frac{550 \times 0.28D^2}{1 + \frac{1}{1600} \times \left(\frac{6250}{D}\right)^2}$

$1250 \times 10^3 \left[1 + \frac{24.41 \times 10^3}{D^2}\right] = 15$

3.05×10^{10} a

$$1 + \frac{24.41\times10^{3}}{D^2} = \left[\frac{154}{1250\times10^{3}}\right] D^2$$

$$\frac{D^2 + 24.41\times10^{3}}{D^2} = 1.23\times10^{-4} D^2$$

$$D^2 + 24.41\times10^{3} = 1.23\times10^{-4} \cdot D^4$$

$$D^2\left(1 + 24.4\right.$$

$$\frac{D^2}{1.23\times10^{-4}\cdot D^2} - \frac{24.41\times10^{3}}{1.23\times10^{-4} D^4}$$

$$\frac{1}{1.23\times10^{-4}} D^2 - 198\,45\times10$$

$$D^2 \cdot -\frac{198.41\times10}{1.23\times10^{-4}}$$

$$\frac{24.41\times10^{3}}{1.23\times10^{-3}} = D^4 - D^2$$

$D = 136$ mm,

$d = 0.8(136) = 10$

Q) A column of circular section is subjected to a load of 120 kN. The load is parallel to the axis but eccentric by an amount of 2.5 mm. The external and internal diameters of the columns are 60 mm & 50 mm respectively. If both the ends of the column are hinged and column is 2.1 m long, then determine the Maximum Stress in column. Take Modulus of Elasticity $E = 200$ GN/m²

Given :

load $P = 120$ kN $= 120\times10^{3}$ N.

eccentricity $e = 2.5$ mm

$D = 60$ mm , $d = 50$ mm.

Length of column $= 2.1$ m $= 2100$ mm

$E = 200$ GN/m²

$= 200\times10^{3}$ N/mm²

$$A = \frac{\pi}{4}(D^2 - d^2) \Rightarrow \frac{\pi}{4}(60^2 - 50^2)$$

$A = 863.93$ mm²

$$I = \frac{\pi}{64}(D^4 - d^4) \Rightarrow \frac{\pi}{64}(60^4 - 50^4)$$

$= 329.37\times10^{3}$ mm⁴

$$Z = \frac{I}{y} \Rightarrow \frac{329.37\times10^{3}}{30} \Rightarrow 10.979\times10^{3}\ \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{P\times e\times \sec\left(\frac{l_e}{2}\times\sqrt{\frac{P}{EI}}\right)}{Z}$$

$$= \frac{120\times10^{3}}{863.93} + \frac{120\times10^{3}\times2.5\times\frac{1}{\cos}\left(\frac{2100}{2}\times\sqrt{\frac{P}{EI}}\right)}{10.979\times10^{3}}$$

$$= 138.900 + \frac{300\times10^{3}\times\frac{1}{\cos}\left[1.4171\right]\text{radian}}{10.979\times10^{3}}$$

$1^\circ = \frac{\pi}{180}$ degree

$= 138.900 + 27.324 \times \sec\left[1.4171\right]\times\frac{1}{}$

$\Rightarrow 81^\circ 1' 3''$

$\sigma_{max} = 317.169$ N/mm²

$d = $

Q) A compound stanchion 3 m long is ... of two channels ISJC 200 (Area = 8554 mm^2) and 250 mm × 10 mm plate revetted one on each flange. Find the maximum load that can be applied e = 20 mm. Take compressive stresses 80 N/mm^2. Take $E = 2\times10^5$ N/mm^2. Assume Hinge ends, Movement of Inertia of the compound section about Y-Y axis may be taken as 4.5×10^7 mm^4

A) length l = 3 m = 3000 mm

Area A = 8554 mm^2

eccentricity e = 20 mm

σ_{max} = Compressive stress = 80 N/mm^2

$E = 2\times10^5$ N/mm^2

$I_{YY} = 4.5 \times 10^7$ mm^4

$$\sigma_{cr} = \frac{\frac{\pi^2 EI}{l^2}}{A} \Rightarrow \frac{\pi^2 \times 2\times10^5 \times 4.5\times10^7}{3000^2}$$

$= 9.86 \times 10^6$ N/mm^2 / 8554

$\Rightarrow 1153.79$

$y_c = \frac{250}{2} = 125$ mm

$r^2 = \frac{I}{A} \Rightarrow 5.260 \times 10^3$

By perry's formula

$$\left(\frac{\sigma_{max}}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_{cr}}\right) = 1.2\left(e\,\frac{y_c}{r^2}\right)$$

$$\left(\frac{80}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{1153.7}\right) = 1.2\left(20 \times \frac{125}{5.26\times10^3}\right)$$

Perry's for

$$\left(\frac{\sigma_{max}}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_{cr}}\right) = 1.2\left(e \cdot \frac{y_c}{r^2}\right)$$

$$\left(\frac{80}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{1153.7}\right) = 0.570$$

$$\left(\frac{80 - \sigma_0}{\sigma_0}\right)\left(\frac{1153.7 - \sigma_0}{1153.7}\right) = 0.570$$

$(80 - \sigma_0)(1153.7 - \sigma_0) = 657.609\,\sigma_0$

$92296 - 80\sigma - 1153.7\sigma_0 + \sigma_0^2 = 657.609\,\sigma_0$

$(\sigma_0^2 - 1233.7\sigma + 91.63\times10^3 = 0)$

$\sigma_0 = 50.12$ N/mm^2 $= 1839.89$ N/mm^2

W.K.T,

$\sigma_0 = \frac{P}{A} \Rightarrow P_0 = \sigma_0 \times A$

$= 50.12 \times 8554$

$= 428.72$ kN.

8

STRUTS :-

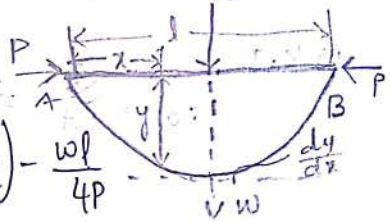
Struts with lateral loads (Beam columns)

columns carrying axial compressive load and Subjected to transverse load is called beam columns.

* Normally the transverse load will be uniformly distributed load but we are considering both the cases i.e., point load as well as UDI

Case i)

Transverse load is the point load acting at the centre



① Max'm Deflection :

$$y_{max} = \frac{w}{2P} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - \frac{wl}{4P}$$

② Max'm BM :

$$M_{max} = \frac{w}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) y_{max}$$

③ Max'm Stress :

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{Z}$$

$$= \frac{P}{A} + \frac{\frac{w}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) y_{max}}{Ar^2}$$

$Z = \frac{I}{y_{max}}$

$r = \sqrt{\frac{I}{A}}$

$I = Ar^2$

$Z = \frac{Ar^2}{y_{max}}$

$$M = \left[-Py + \frac{w}{2}x\right]$$ (for concave)

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \qquad \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left[-Py + \frac{w}{2}x\right]}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI} + \left(\frac{w}{2}x\right)\frac{1}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{\left(\frac{w}{2}x\right)}{EI} \times \frac{P}{P}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{wx}{2P} \times \frac{P}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a$$

$$\frac{d^2y}{dx^2} + \sqrt{\frac{P}{EI}} \cdot y = \sqrt{\frac{P}{EI}} \cdot \frac{wx}{2P}$$

$$y = A\sin\sqrt{\frac{P}{EI}} \cdot x + B\cos\sqrt{\frac{P}{EI}}\, x + \frac{wx}{2P}$$

Boundary conditions :

at $x = 0$, $y = 0$

$$0 = A\sin\sqrt{\frac{P}{EI}} \cdot 0 + B \cdot \cos(0) + \frac{w(0)}{2P}$$

$$B = 0$$

at $x = l/2$, $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\, x - B\sin\sqrt{\frac{P}{EI}}\, x + \frac{wx}{2P}$$

$$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2}\right) - B\sin\sqrt{\frac{P}{EI}}\left(\frac{l}{2}\right) + \frac{w}{2P}(1)$$

$$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2}\right)\sqrt{\frac{P}{EI}} + \frac{w}{2P}$$

$$\frac{dy}{dx} = A\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2}\right)\sqrt{\frac{P}{EI}} + \frac{w}{2P}$$

$$0 = A\cos\sqrt{\frac{P}{EI}}\left(\frac{l}{2}\right)\sqrt{\frac{P}{EI}} + \frac{w}{2P}$$

$$A = \frac{\frac{-w}{2P}}{\cos\left(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2}\right) \cdot \sqrt{\frac{P}{EI}}}$$

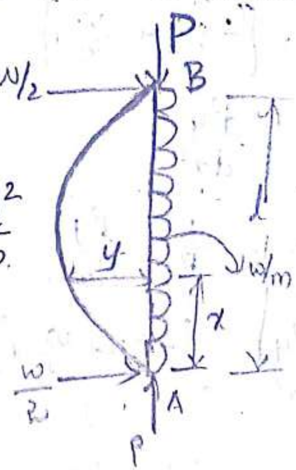
$$= \frac{-w}{2P} \cdot \sqrt{\frac{EI}{P}} \cdot \sec\left(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2}\right)$$

$$y = \frac{-w}{2P}\sqrt{\frac{EI}{P}} \cdot \sec\left(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2}\right) \cdot \sin\left(\sqrt{\frac{P}{EI}}\, x\right) + \frac{wx}{2P}$$

At $x = \frac{l}{2}$, $y = y_{max}$

$$y_{max} = \frac{-w}{2P}\sqrt{\frac{EI}{P}} \tan\left(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2}\right) + \frac{wl}{4P}$$

Case ii) strut subjected to UDL as transverse load



i) Max'm Deflection:

$$y_{max} = \frac{w \times EI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] - \frac{wl^2}{8P}$$

ii) Mmax BM:-

$$M_{max} = \frac{w \times EI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

iii) σ_{max}

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{Z}$$

Q) Determine the Maximum stress induced in the cylindrical ~~strut~~ steel strut of length 1.2 m and dia 30 mm. This strut hinged at both ends and subjected to an axial thrust of 20 kN at its ends and transverse point load 1.8 kN at the centre. Take E = 208 GN/m²

Given:

length $l = 1.2\ m = 1200\ mm$

dia $= 30\ mm$

both ends are hinged.

$P = 20\ kN = 20 \times 10^3\ N$

$W = 1.8\ kN = 1.8 \times 10^3\ N$

$E = 208 \times 10^3\ N/mm^2$

Max'm stress.

$$\sigma_{max} = \frac{P}{A} + \frac{\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) y_{max}}{Ar^2}$$

$A = \frac{\pi}{4} \times d^2 \Rightarrow 706.858\ mm^2$

$y_{max} = \frac{d30}{2} = 15\ mm$

$I = \frac{\pi}{64} d^4 \Rightarrow 39.760 \times 10^3\ mm^4$

$$\sigma_{max} = \frac{20 \times 10^3}{706.858} + \frac{\frac{1.8 \times 10^3}{2}\sqrt{\frac{208 \times 10^3 \times 39.760 \times 10^3}{20 \times 10^3}} \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}{706.858\ mm^2}$$

$$\sigma_{max} = 28.294 + \frac{3.809 \times 10^6 \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) y_{max}}{Ar^2}$$

$$\sigma_{max} = 28.294 + \frac{38.09 \times 10^6 \times \tan(0.9330) \times \frac{180}{\pi} \times 15}{A \times r^2}$$

$r^2 = \frac{I}{A}$

$r^2 = 56.248$

$$\sigma_{max} = 28.294 + \frac{38.09 \times 10^6 \tan(0.9330) \times \frac{180}{\pi} \times 15}{706.858 \times 56.248}$$

$\sigma_{max} = 28.294 + 13.408 \times 10^3$

$\sigma_{max} = 13.436 \times 10^3\ N/mm^2$

Q) Determine the maximum stress induced in a horizontal strut of length 2.5 m and of rectangular cross section 40 mm x 80 mm. when it carries an axial thrust of 100 kN. and a verticle load of 6 kN/m length. The strut having pin joints at its ends, Take. $E = 208\ GN/m^2$

$l = 2.5\ m$, $b = 0.04\ m$, $d = 0.08\ m$

$P = 100\times10^3\ N$

$W = 6\times10^3\ N/m$

$E = 208\times10^9\ N/m^2$

$I = \frac{bd^3}{12} = \frac{40\times80^3}{12}$

$= 1.706\times10^6\ mm^4$

$= 1.706\times10^{-6}\ m^4$

$M_{max} = \omega \times \frac{EI}{P}\left[\sec\left(\frac{l}{2}\times\sqrt{\frac{P}{EI}}\right)-1\right]$

$= 6\times10^3 \times \frac{208\times10^9\times1.706\times10^{-6}}{100\times10^3}\left[\sec\left(\frac{2.5}{2}\right)\times\sqrt{\frac{100\times10^3}{208\times10^9\times1.706\times10^{-6}}}-1\right]$

$= 1.248\times10^{10}\left[\sec\left(\frac{2.5}{2}\times0.5308\right)-1\right]$

$1.248\times10^{10}\times\left[\sec\left(0.663\times\frac{180}{\pi}\right)-1\right]$

$1.248\times10^{10}\left[\sec(37.958')-1\right]$

$(1.26-1)$

UNIT-III Combined direct & Bending stresses

Direct stress :-

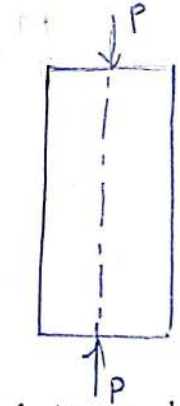
Direct stress is produced in a body when it is Subjected to an axial load (Tensile or compressive)

Bending stress :-

Bending stress is produced when it is subjected to a Bending Moment which is either due to eccentric (or) direct Bending Moment.

combined direct and Bending stresses :-

Consider a column Subjected to a compressive load 'P' along the axis of the column



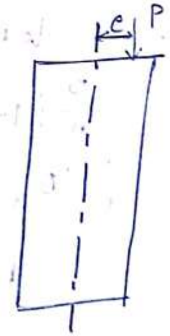
Let σ_0 = Intensity of direct stress

A = Area of cross section

P = Load acting on column

$$\boxed{\sigma_0 = \frac{P}{A}}$$

Now, consider the case of a column Subjected to eccentric loading whose line of action is at a distance of 'e' from the axis of the column as shown in the figure



Let P = eccentric Load on column

e = eccentricity of the Load

σ_0 = Direct stress

σ_b = Bending stress

b = width of the column

Moment of force M = Force $\times$ Lever distance

$$M = P \times e$$

$$\sigma_0 = \frac{P}{A} \text{ (uniform throughout the section)}$$

The Bending stress σ_b due to moment at any point of the column section is given by

$$\frac{M}{I} = \frac{f}{y} \Rightarrow \sigma_b = \frac{M}{I} \times y$$

Note :- Since eccentricity is taken from Y-Y axis, so Moment of Inertia has to consider about Y-Y axis

i.e. $I_{y\text{-}y} = \frac{DB^3}{12}$; $y = \frac{b}{2}$

$$\frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \sigma_b = \frac{M}{I} \times y$$

$$\sigma_b = \frac{P \times e}{\frac{db^{3}}{12}} \times \frac{b}{2}$$

$$\sigma_b = \frac{6P \times e}{db \times b} \Rightarrow \frac{6P \times e}{Ab}$$

$$\sigma_{max} = \sigma_0 + \sigma_b = \frac{P}{A} + \frac{6Pe}{Ab} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$$

$$\sigma_{min} = \sigma_0 - \sigma_b = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$ → For I_{yy} we take d

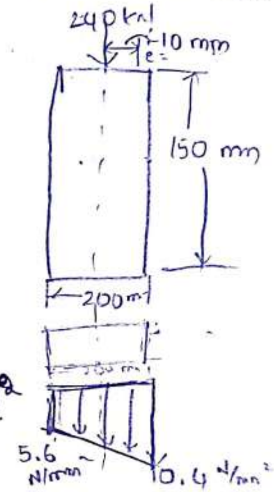
for no tension condition, $\sigma_{min} \geq 0$;

$$\frac{P}{A}\left(1 - \frac{6e}{b}\right) \geq 0$$

$$\because \frac{P}{A} \neq 0, \left(1 - \frac{6e}{b}\right) \geq 0$$

$$1 > \frac{6e}{b} \qquad \therefore \boxed{e < \frac{b}{6}}$$

Q) A rectangular column of width 200 mm and of thickness 150 mm carries a point load of 240 kN at an eccentricity of 10 mm. Determine the Maximum and Minimum stresses on the section.



width b = 200 mm

thickness t = 150 mm

point load $P = 240 \times 10^3$ N

$e = 10$ mm

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$$

$$A = b \times t = 200 \times 150 = 30 \times 10^3 \text{ mm}^2$$

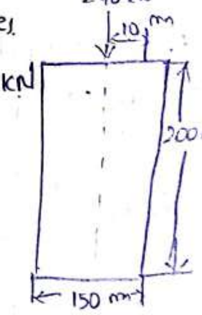
$$\sigma_{max} = \frac{240 \times 10^3}{30 \times 10^3}\left(1 + \frac{6 \times 10}{200}\right)$$

$$\sigma_{max} = 10.4 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right] = \frac{240 \times 10^3}{30 \times 10^3}\left(1 - \frac{6 \times 10}{200}\right)$$

$$\sigma_{min} = 5.6 \text{ N/mm}^2$$

Q) A rectangular column of width 150 mm, depth 200 carries a point load of 240 kN with e of 10 mm. Determine the Max'm & Min'm stresses



b = 150 mm; d = 200 mm e = 10 mm, P = 240 kN

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right] = \frac{240 \times 10^3}{200 \times 150}\left[1 + \frac{6 \times 10}{150}\right]$$

$$\sigma_{max} =$$

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right] = \frac{240 \times 10^3}{200 \times 150}\left[1 - \frac{6 \times 10}{150}\right]$$

In continuation to problem 1, if minimum stress is zero find the eccentricity and also the max'm stress

$\sigma_{min} = 0$

$\sigma_{min} = \frac{P}{A}\left[1 \mp \frac{6e}{b}\right]$ $\qquad \frac{P}{A}\left[1 - \frac{6e}{b}\right]$ $\qquad 0 = \frac{240\times10^3}{30\times10^3} - 0.24e$

$5.6 \qquad 5.6 = \frac{240\times10^3}{30\times10^3}\left[1 \mp \frac{6\times e}{200}\right]$

$5.6 = 8\left[1 \mp \frac{6e}{200}\right]$

$(5.6 = 8 + 0.16e)$

$5.6 = 8 - \frac{48e}{200}$ $\quad 8 - 0.24e$

$e = 33.33$

$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$

$10.4 = \frac{240\times10^3}{30\times10^3}\left[1 + \frac{6e}{200}\right]$

$10.4 = 8 + \frac{48e}{200}$ $\qquad 8 + 0.24e = 10.4$ $\qquad 10.4 - 8$

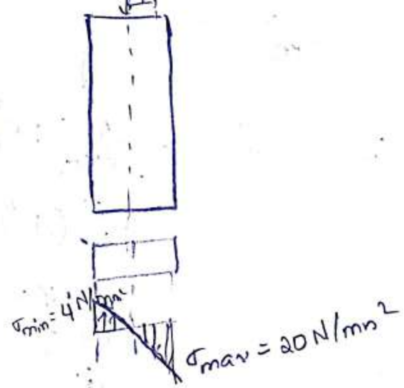
$e = 15.99$ N/mm²

Q) If eccentricity = 50 mm in the above mention prblm (3) Plot the stresses along width of the Section

$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$ $\qquad \sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$

$= \frac{240\times10^3}{30\times10^3}\left[1 + \frac{6\times50}{200}\right]$ $\qquad \sigma_{min} = -4$ N/mm²

$= 20$ N/mm²



Q) The line of thrust in a compression testing Machine Specimen 15 mm dia is parallel to the axis of the specimen but is displaced from it. Calculate the distance of the line from the axis when the Max'm stress is 20% greater than the Mean stress on the Normal Section.

dia = 15 mm , e = ?

σ_{max} = 20% greater than mean stress

$\sigma_{max} = 1.2\,\sigma_{mean}$

$\sigma_{max} = \sigma_0 + \sigma_b$

$= \frac{P}{A} + \frac{M}{Z} = \frac{P}{A} + \frac{Pe}{Z}$

$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$

$= \frac{(\sigma_0 + \sigma_b) + (\sigma_0 - \sigma_b)}{2}$

$= \frac{2\sigma_0}{2} = \sigma_0 = \frac{P}{A}$

$\sigma_{max} = 1.2\,\sigma_{mean}$

$\frac{P}{A} + \frac{Pe}{Z} = 1.2 \cdot \frac{P}{A} \Rightarrow \frac{1}{A} + \frac{e}{Z} = \frac{1.2}{A}$

$$Z = \frac{\pi d^3}{32} = \frac{\pi (15)^3}{32} = 331.33 \text{ mm}^3$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (15)^2}{4} = 176.71 \text{ mm}^2$$

$$\frac{1}{A} + \frac{e}{Z} = \frac{1.2}{A}$$

$$\frac{1}{176.71} + \frac{e}{331.33} = \frac{1.2}{176.71}$$

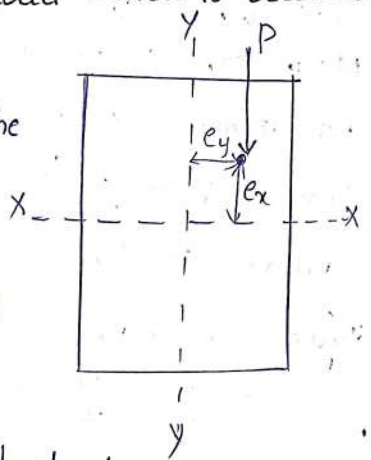
$$e = 0.375 \text{ mm}$$

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}(d^4)}{\frac{\pi}{4}(d^2)}$$

$$= \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \frac{\pi d^3}{32}$$

7/04/23

Resultant Stress when a column of Rectangular Section is subjected to a load which is eccentric to both the axis.



Let p = eccentric Load on the Column

e_x = eccentricity of load about x-x axis (or) distance from point to the x-axis

e_y = eccentricity of load about Y-Y axis

b = width

d = depth

σ_0 = direct stress

σ_{bx} = bending stress due to eccentricity e_x

σ_{by} = bending stress due to eccentricity. e_y.

M_x = moment of load about x-axis

M_y = Moment of load about Y-axis.

I_{xx} = Moment of Inertia w.r.t x-axis $= \frac{bd^3}{12}$

I_{yy} = Moment of Inertia w.r.t. Y-axis $= \frac{db^3}{12}$

(i) Direct stress $\sigma_0 = \frac{P}{A}$

(ii) σ_b due to $e_y = \sigma_{by} = \frac{M_y}{I_{y\text{-}y}} \times x$. $\left[x = -\frac{b}{2}, +\frac{b}{2}\right]$

(iii) σ_b due to $e_x = \sigma_{bx} = \frac{M_x}{I_{xx}} \times y$ $\left[y = \frac{-d}{2}, +\frac{d}{2}\right]$

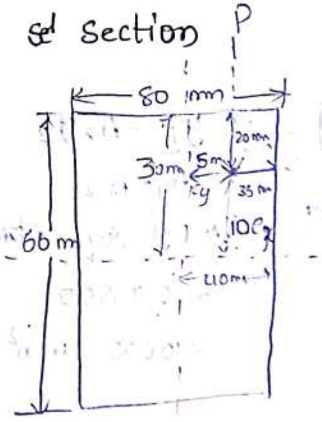
$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_{max/min} = \frac{P}{A} \pm \frac{M_y}{I_{yy}} \times x \pm \frac{M_x}{I_{x\text{-}x}} \times y$$

$$\sigma_{max}\,(comp) = \frac{P}{A} + \frac{M_y}{I_{y\text{-}y}} \times x + \frac{M_x}{I_{x\text{-}x}} \times y.$$

$$\sigma_{min}\,(Ten) = \frac{P}{A} - \frac{M_y}{I_{y\text{-}y}} \times x - \frac{M_x}{I_{x\text{-}x}} \times y.$$

Q) A short column of Rectangular c/s 80 mm × 60 carries a load of 40 kN at a point 20 mm from the longer side. and 35 mm from the shorter side. Determine the maximum compress and Tensile stresses in the sd section



Given:

$L = 80$ mm

$d = 60$ mm

$P = 40 \text{ kN} = 40 \times 10^3$ N.

$e_x = 10$ mm, $e_y = 5$ mm

$$I_{xx} = \frac{bd^3}{12} = 1.44 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{60 \times 80^3}{12} = 2.56 \times 10^6 \text{ mm}^4$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_y}{I_{yy}} \times x + \frac{M_x}{I_{x-x}} \times y$$

$$M_y = 40\times10^3 \times 5 \text{ mm} = 200\times10^3 \text{ N·m}$$

$$\sigma_{max} = \frac{40\times10^3}{80\times60} + \frac{200\times10^3}{2.56\times10^6}\times\frac{80}{2} + \frac{200\times10^3\times 9}{1.44\times10^6}\times\frac{60}{2}$$

$$= 19.79 \text{ N/mm}^2$$

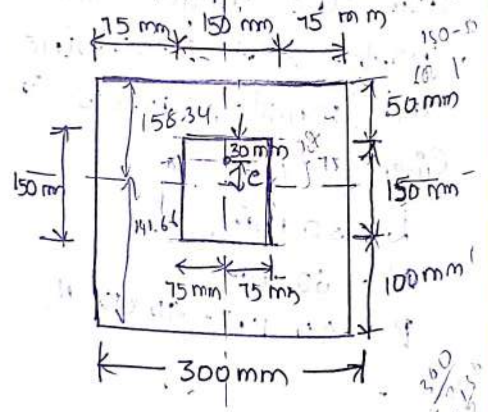
$$\sigma_{min} = \frac{40\times10^3}{80\times60} - \frac{200\times10^3}{2.56\times10^6}\times\frac{80}{2} - \frac{200\times10^3\times10}{1.44\times10^6}\times\frac{60}{2}$$

$$= 8.333 \text{ to } -3.125 \text{ N/mm}^2$$

Q) A short column has a square Section 300 mm x 300 mm with a Square hole of 150 mm x 150 mm as shown in the figure. It carries an eccentric Load of 1800 KN. Loaded as shown in the figure. Determine the max'm compressive and Tensile stress across the Section

not Symmetry about x-axis

when it is a hollow section (–)
when " " " Solid Section (+)



$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

A_1 = Area of sq. column.

$= 300 \times 300$

$= 90000 \text{ mm}^2$

A_2 = area of square hole

$= 150 \times 150 \text{ mm}^2 = 22500 \text{ mm}^2$

$$y_1 = \frac{300}{2} = 150 \text{ mm}, \quad y_2 = 100 + \frac{150}{2} = 175 \text{ mm}$$

$$\bar{y}_b = \frac{90000 \times 150 - 22500 \times 175}{90000 - 22500}$$

$$\bar{y}_b = 141.66 \text{ mm}$$

$$y_t = 300 - 141.66 = 158.34$$

$$e = 158.34 - 50 - 30 = 78.34 \text{ mm}$$

$$\sigma_0 = \frac{P}{A} = \frac{1800\times10^3 \text{ N}}{A_1 - A_2} = \frac{1800\times10^3}{9000-22500} = 26.66 \text{ N/}$$

$$\sigma_b = \frac{M}{z} = \frac{P\times e}{I/y}$$

$(I = I_1 - I_2)$

$$I_1 = \frac{300^4}{12} = 675\times10^6 \text{ mm}$$
$$I_2 = \frac{150^4}{12} = 42.1875\times10^6$$

$$I = I_1 - I_2$$

$$I = \frac{bd^3}{12} + A(\bar{y}_b)^2 \qquad I_1 = (y_b - y_1)^2$$

$$I_1 = \frac{300\times300^3}{12} + 90\times10^3(141.67-150)^2 \qquad I_2 = (y_b - y_2)^2$$

$$= 681.245\times10^6 \text{ mm}^4$$

$$I_2 = \frac{150\times150^3}{12} + 22.5\times10^3(141.67-175)^2$$

$$= 67.182\times10^6 \text{ mm}^4$$

$$I = I_1 - I_2 = 614.065\times10^6 \text{ mm}^4$$

$$Z_{min} = \frac{I}{y_{max}} = \frac{614.065 \times 10^6}{y_n\ 158.34} \qquad \therefore \sigma_{bmax} = \frac{M_{max}}{Z_{min}}$$

$$Z_{min} = 3.87 \times 10^6 \text{ mm}^3$$

$$Z_{max} = \frac{I}{y_{min}} = \frac{614.065 \times 10^6}{141.66} = 4.33 \times 10^6 \text{ mm}^3$$

$$\sigma_{max} = \sigma_0 \pm \sigma_{bmax}$$

$$= 26.67 + \frac{P \times e}{Z_{min}}$$

$$= 26.67 + \frac{1800 \times 10^3 \times 78.34}{3.87 \times 10^6}$$

$$= 26.67 + 36.43$$

$$= 63.107 \text{ N/mm}^2$$

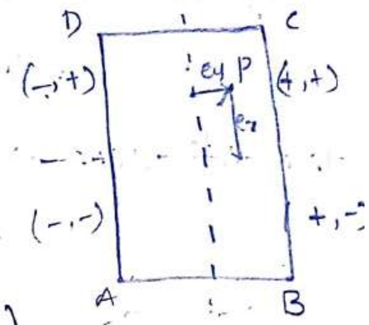
$$\sigma_{min} = \sigma_0 \mp \sigma_{b\,min}$$

$$= 26.67 - \frac{P \times e}{Z_{max}}$$

$$= 26.67 - \frac{1800 \times 10^3 \times 78.34}{4.33 \times 10^6}$$

$$\left\{ \begin{array}{l} 26.67 - (-5.89) \\ = 32.56 \text{ N/mm}^2 \end{array} \right\} = 26.67 - (32.56) = -5.89 \text{ N/mm}^2$$

Note :- Stress in four corners



$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{I} \times (y) + \frac{M}{I}(x)$$

$$\sigma_{max} @ A = \frac{P}{A} + \frac{M}{I}(y) + \frac{M}{I}(-x)$$

$$\sigma_{max} @ B = \frac{P}{A} + \frac{M}{I}(-y) + \frac{M}{I}(x)$$

$$\sigma_{max} @ C = \frac{P}{A} + \frac{M}{I}(y) + \frac{M}{I}(x)$$

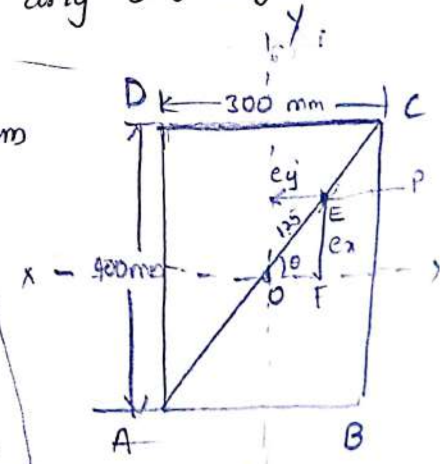
$$\sigma_{max} @ D = \frac{P}{A} + \frac{M}{I}(+y) + \frac{M}{I}(-x)$$

Q) A column is rectangular in cross section of 300 mm x 400 mm in dimensions. The column carries an eccentric point load of 360 kN at a distance of quarter one diagonal length from a corner. Calculate the diagonal stresses at all four corners. Draw stress distribution diagrams for any two adjacent sid

Given

$b = 300$ mm, $d = 400$ mm

$P = 360$ kN $= 360 \times 10^3$ N

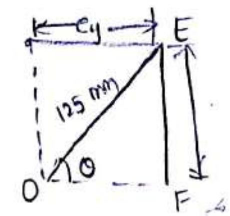


AC length = 500 mm

Also OE = EC =>

$\frac{500}{4} = 125$ mm = OE = EC

The eccentric is acting at point 'E', where distance EC = quarter length of diagonal AC. Now diagonal AC length =



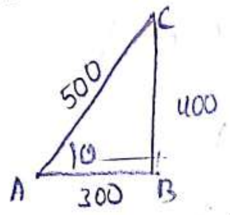
$$\cos\theta = \frac{OF}{OE}$$

$$\sin\theta = \frac{EF}{OE}$$

$e_y = OF = OE\cos\theta$

$e_x = EF = OE\sin\theta$

from Δ^{le} $\angle CAB \Rightarrow \cos\theta = \frac{300}{500} = \frac{3}{5}$



$\sin\theta = \frac{400}{500} = \frac{4}{5}$

$e_y = OF = OE\cos\theta = 125 \times \frac{3}{5} = 75$ mm

$e_x = EF = OE\sin\theta = 125 \times \frac{4}{5} = 100$ mm

Area $= 300 \times 400 = 120 \times 10^3$ mm^2

$I_{xx} = \frac{bd^3}{12} = \frac{300 \times 400^3}{12} = 1.6 \times 10^9$ mm^4

$I_{yy} = \frac{db^3}{12} = \frac{400 \times 300^3}{12} = 900 \times 10^6$ mm^4

$M_x = 360 \times 10^3 \times 100 = 36 \times 10^6$ N·mm

$M_y = 360 \times 10^3 \times 75 = 27 \times 10^6$ N·mm

$y = \frac{d}{2} = \frac{400}{2} = 200$ mm

$x = \frac{b}{2} = \frac{300}{2} = 150$ mm

σ_{max} @ A $= \frac{P}{A} + \frac{M_x}{I_{xx}}(-y) + \frac{M_y}{I_{yy}}(-x)$

$\frac{360 \times 10^3}{120 \times 10^3} + \frac{36 \times 10^6}{1.6 \times 10^9} \times (-200) + \frac{27 \times 10^6}{900 \times 10^6}(-150)$

$3 + 2.25 \times 10^{-3}(-200) + 0.04(-150)$

σ_{max} @ A $= -6$ N/mm^2 (T)

σ_{max} @ B $= \frac{P}{A} + \frac{M}{I}(-y) + \frac{M}{I}(x)$

σ_{max} @ B $= \frac{360 \times 10^3}{120 \times 10^3} + \frac{36 \times 10^6}{1.6 \times 10^9}(-200) + \frac{27 \times 10^6}{900 \times 10^6}(150)$

$= 3$ N/mm^2 (c)

σ_{max} @ C $= \frac{P}{A} + \frac{M}{I}(y) + \frac{M}{I}(x)$

$= \frac{360 \times 10^3}{120 \times 10^3} + \frac{36 \times 10^6}{1.6 \times 10^9}(200) + \frac{27 \times 10^6}{900 \times 10^6}(150$

$= 12$ N/mm^2 (c)

σ_{max} @ D $= \frac{P}{A} + \frac{M}{I}(y) + \frac{M}{I}(-x)$

$= \frac{360 \times 10^3}{120 \times 10^3} + \frac{36 \times 10^6}{1.6 \times 10^9}(200) + \frac{27 \times 10^6}{900 \times 10^6}($

$= 3$ N/mm^2 (c)

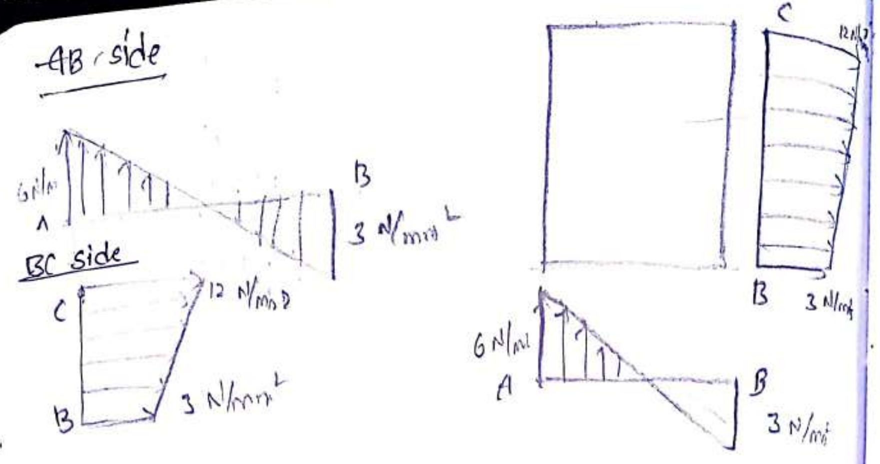
Stress Distribution for AB and BC (i.e. two adjacent sides).

at point A 6 N/mm^2 [T] whereas @ B the resultant stress is 3 N/mm^2 [c].

Take AE = 6 N/mm^2 and BF = 3 N/mm^2.

Join E and F

For side BC, the resultance stress at point B 3 N/mm^2 (c) whereas @ point C the resultant stress 12 N/mm^2 [c]



Core (or) Kernal of a Section

It is the area within which the line of action of eccentric load must cut the c/s, If the stresses are not tensile in any part of the section.

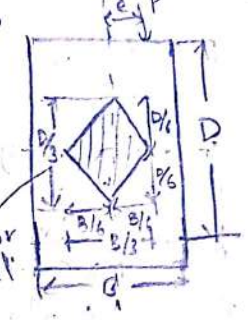
Note:- The cement concrete columns are weak in Tension hence the load must be applied on these columns in such a way that there is no tensile stress anywhere in the section but when an eccentric load is acting on a column it produces direct stress as well as bending stress. The resultant stress at any point in the section is the algebraic sum of the direct stress and Bending stress.

· Kernal of a Rectangular Section:-

Let 'P' eccentric Load acting on the column

e = eccentricity of load, A = Area of Section

For No tension condition to develop



Core or kernal

$\sigma_{min} \geq 0$

$(\sigma_0 - \sigma_b) \geq 0$

$\frac{P}{A} - \frac{M}{Z} \geq 0$

$\frac{P}{A} - \frac{P \times e \times y}{I} \geq 0$

$\frac{P}{A} - \frac{P \times e \times D}{\frac{BD^3}{12_6} \times 2} \Rightarrow \frac{P}{A} - \frac{6 P \times e}{(B \times D) D} \geq 0$

$\frac{P}{A} - \frac{6Pe}{AD} \geq 0$

$\frac{P}{A}\left[1 - \frac{6e}{D}\right] \geq 0 \qquad \therefore \frac{P}{A} \neq 0$

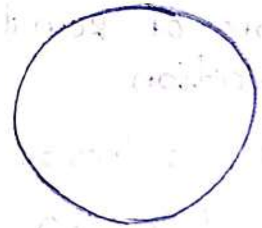
$1 - \frac{6e}{D} \geq 0$

$\frac{6e}{D} \geq 1 \Rightarrow e \leq \frac{D}{6}$

Similary $e \leq \frac{b}{6}$

Hence this is also known as middle third.

Kernal of a circular Section



Let P eccentric Load acting on column

e = ecentricity of load

A = Area of section

For no tension condition to develop

$\sigma_{min} \geq 0$

$(\sigma_0 - \sigma_b) \geq 0$

$\frac{P}{A} - \frac{M}{Z} \geq 0$

$\frac{P}{A} - \frac{P \times e \times y}{I} \geq 0$

$\frac{P}{A} - \frac{P \times e \times D}{\frac{\pi}{64} D^4 \times 2} \Rightarrow \left\{ \frac{P}{A} - \frac{P \times e \times D}{\frac{\pi D^2}{4}\left[\frac{D^2}{18}\right] \times 2} \right\}$

$\frac{P}{A} - \frac{P \times e \times D}{\frac{\pi D^{4\,3}}{\cancel{64}\,32} \times \cancel{2}}$ $\quad \frac{\pi D^3}{32}$ $\quad 32 P \times e \times y$

$\frac{P}{A} - \frac{P \times e \times \cancel{2}}{\frac{\pi D^2}{32}} \Rightarrow \frac{P}{A} - \frac{P \times e}{\frac{\pi D^2}{4} \times \left(\frac{D}{8}\right)}$

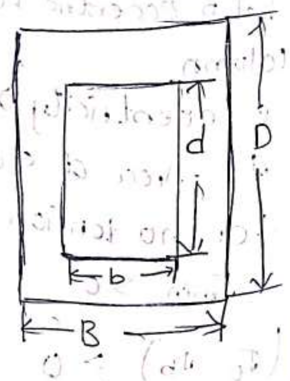
$\frac{P}{A} - \frac{P \times e}{A/8}$

$\frac{P}{A}\left(1 - \frac{8e}{D}\right)$

$1 - \frac{8e}{D} \geq 0$

$\therefore \frac{8e}{D} \leq 1 \quad e \leq \frac{D}{8}$

Core or kernal of a Hollow Rectangular Section



For no tension condition,

$\sigma_{min} \geq 0$

$(\sigma_0 - \sigma_b) \geq 0$

$\frac{P}{A} - \frac{M}{Z} \geq 0$

$\frac{P}{A} - \frac{P \times e \times y}{I} \geq 0$

$\frac{P}{A} - \frac{P \times e}{\frac{BD^3 - bd^3}{\cancel{12}\,6}} \times \frac{D}{2}$

$\frac{P}{A} - \frac{6P \times e \times D}{(BD^3 - bd^3)} \geq 0$

$\frac{P}{A} - \frac{6PeD}{BD^3 - bd^3} \geq 0$

$\frac{P}{A} - \frac{6PeD}{BD^3 - bd^3} \geq 0$

$\frac{P}{A} \geq \frac{M}{Z}$

$\frac{P}{A} = \frac{P \times e}{Z}$

$e \leq \frac{Z}{A}$

$Z = \frac{I}{y}$

$= \frac{BD^3 - bd^3}{6\,\cancel{12} \times D/2}$

$Z = \frac{BD^3 - bd^3}{6D}$

$A = BD - bd$

$\frac{Z}{A} = \frac{BD^3 - bd^3}{6D(BD - bd)}$

$\boxed{e \leq \frac{BD^3 - bd^3}{6D(BD - bd)}}$

Core or kernal of a Hollow circular Section

For no tension Condition to develop

$\sigma_{min} \geq 0$

$\sigma_0 - \sigma_b \geq 0$

$\frac{P}{A} - \frac{M}{Z} \geq 0$

$\frac{P}{A} - \frac{P \times e}{Z} \geq 0$

$Z = \frac{\pi[D^4 - d^4]}{\cancel{64}\,32} \times \frac{\cancel{2}}{D}$

$Z = \frac{\pi[D^4 - d^4]}{32D}$

$\frac{P}{A} \geq \frac{P \times e}{Z}$

$e \leq \frac{Z}{A} \quad \because A = \frac{\pi}{4}(D^2 - d^2)$

$Z = \pi$

$Z = \frac{\pi}{D}[D^4 - d^4]$

$\frac{Z}{A} = \frac{\frac{\pi}{32D}[D^4 - d^4]}{\frac{\pi}{4}[D^2 - d^2]}$

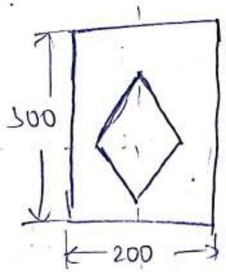
$\frac{Z}{A} = \frac{1}{8D}\left[\frac{(D^2 + d^2)(D^2 - d^2)}{D^2 - d^2}\right]$

$e \leq \frac{D^2 + d^2}{8D}$

Q) Draw neat sketches of kernal of the following cross section i) Rectangular section 200 x 300 mm

ii) Hollow circular cylinder with external dia 200 mm and thickness 50 mm.

iii) Square with 400 cm^2.



i) $\sigma_{min} \geq 0$

$(\sigma_0 - \sigma_b) \geq 0$

$$\frac{P}{A} - \frac{P \times e \times y}{I} \geq 0$$

$$\frac{P}{B \times D} - \frac{P \times e \times \frac{D}{2}}{\frac{BD^3}{12}} \geq 0$$

$$\frac{P}{60 \times 10^3} - \frac{P \times e \times 100}{200 \times 10^6} \geq 0$$

$$\frac{1}{60 \times 10^3} \geq \frac{e \times 100}{200 \times 10^6}$$

$$\frac{1}{60 \times 10^3} = \frac{e \times 100}{200 \times 10^6}$$

$e = 33.33$

$$\frac{d}{6} = \frac{300}{6} = 50$$

$y - \frac{b}{2}$

Hollow circular

$-\sigma_0 - \sigma_b \geq 0$

$$\frac{P}{A} - \frac{P \times e}{Z} \geq 0$$

$$Z = \frac{\pi (D^4 - d^4)}{32 D} = 736.310 \times 10^3$$

$$A = \frac{\pi}{4}(D^2 - d^2) = 23.561 \times 10^3$$

$$\frac{1}{A} = \frac{e}{Z}$$

$$e \leq \frac{Z}{A}$$

$e \leq 31.251$

$$\frac{Z}{A} \left[\frac{\frac{\pi}{32} D (D^4 - d^4)}{\frac{\pi}{4}(D^2 - d^2)} \right]$$

$$\frac{Z}{A} = \frac{1}{8D} \left(\frac{(D^2 + d^2)(D^2 - d^2)}{D^2 - d^2} \right)$$

$$e \leq \frac{D^2 + d^2}{8D}$$

---

iii) $(\sigma_0 - \sigma_b) \geq 0$

$$\frac{P}{A} - \frac{M}{Z} \geq 0$$

$$\frac{P}{A} - \frac{P \times e \times y}{I} \geq 0$$

$$\frac{1}{A} = \frac{e \times y}{\frac{4000^4}{12}}$$

$$\frac{1}{4000 \times 400} = \frac{e \times 2000}{2.133 \times 10^{13}}$$

$e = 66.56$

Chimneys

→ Chimneys are tall structures subjected to horizontal wind pressure

→ Bending Moment at the base due to wind produ the bending stress.

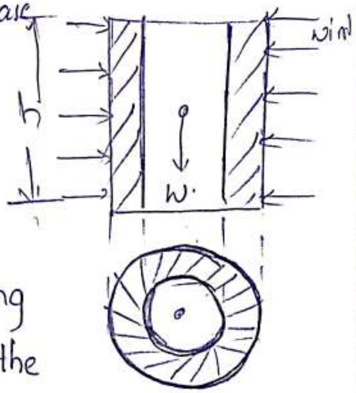
→ Due to self weight. direct stress is produced

Step 1: Due to self weight direct stess σ_0

$$\sigma_0 = \frac{P}{A} = \frac{\text{weight of chimney}}{\text{Area of section @ base}}$$

Step 2: Bending Moment at base due to wind force. produce Bending stress

$$\sigma_b = \frac{M}{Z}$$



Step 3: The wind force (F) acting in the horizontal direction on the surface of chimney is given by

$F = K \times P \times A$ where,

K = coeff. of wind resistance.

P = wind pressure

Case - i)

If $K = 1$, for rectangular c/s section

Case ii)

If $K = \frac{2}{3}$, for circular c/s

A = Area projected to wind pressure

case i); If it is a rectangular c/s $A = b \times h$

case ii): $D \times h$; for circular cross section.

Step 4: Moment of force at the base of a chimney $M = F \times \frac{h}{2}$

Q) Determine the maximum and minimum stresses at base of a hollow circular chimney. of height 20 m., with external dia 4 m. and Internal dia 2 m. The chimney is subjected to horizontal wind pressure intensity 1 kN/m². The specific weight of the material of chimney is 22 kN/m²

Sol: Given.

$h = 20$ m

$D = 4$ m.

$d = 2$ m

$P = 1$ kN/m²

Specific wt. of chimney = 22 kN/m²

$$sp.\,w = \frac{w}{vol}$$

Area

$w = vol \times sp.\,wt$

$w = A \times l \times sp.\,wt$

$$= \frac{\pi}{4}[D^2 - d^2] \times 22 \times 10^3 \times 20$$

$w = 4.146 \times 10^6$

S-1) $\sigma_0 = \frac{W}{A}$

$A = \frac{\pi}{4}\left[D^2-d^2\right] \Rightarrow \frac{\pi}{4}\left(4^2-2^2\right) = 9.42\ m^2$

$Sp\ wt = \frac{wt}{Vol}$

$W = sp.wt \times vol.$

$\cancel{(W = 440\ kN/m^2)}$

$W = 4.144 \times 10^6\ N$

$\sigma_0 = \frac{W}{A} = \frac{4.144 \times 10^6}{9.42} = 440\ kN/m^2$

S-2) $\sigma_b = \frac{M}{Z}$

$Z = \frac{I}{y} = \frac{\frac{\pi}{64}(D^4-d^4)}{D/2} = \frac{\pi(D^4-d^4)}{32D}$

$= \frac{\pi(4^2-2^4)}{32 \times 4} = 5.89\ m^3$

$M = F \times \frac{h}{2}$

$F = K \times P \times A \qquad K = \frac{2}{3}$

$= \frac{2}{3} \times 1 \times D \times h$

$= \frac{2}{3} \times 1 \times 4 \times 20$

$F = 53.33\ kN$

$M = 53.33 \times \frac{20}{2}$

$M = 533.33\ kN.m$

$\sigma_0 = \frac{M}{Z} = \frac{533.33}{5.89} =$

$\sigma_b = 90.54\ kN/m^2$

Max stress $= \sigma_0 + \sigma_b$

$= 440 + 90.54$

$= 530.54\ kN/m^2$

Min stress $= \sigma_0 - \sigma_b$

$= 440 - 90.54$

$= 349.46\ kN/m^2$

Q) A masonry dam of rectangular c/s 10 m height and 5 m wide as water up to the top on its one side. If the weight density of masonry is 21.582 kN/m³. Find

i) pressure force due to water per meter length of dam.

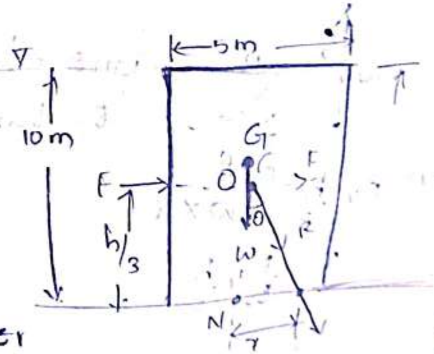
ii) Resultant force & the point at which it cuts the base of dia.

iii) Find σ_{max} & σ_{min}

Given

$H = 10\ m,\ b = 5\ m,\ h = 10\ m$

wt of masonry $= 21.58$ kN.



i) pressure force due to water per m length

$F = w \times A \times \bar{h}$

$= w_0 \times (h \times 1m) \times \frac{h}{2} = \frac{wh^2}{2}$ N

hydrostatic law $\boxed{P = wh} \Rightarrow \frac{F}{A} = wh \Rightarrow F = wA\bar{h}$

$w = \rho g$ · $g = 9.81$, $\rho = 1000$

$F = \frac{wh^2}{2} = \frac{9.81 \times 1000 \times (10)^2}{2} = 490.5$ N

$F = 490.5$ N

ii) wt of dam $= w \times vol = w \times (A \times length)$

$\frac{W}{vol} = sp.wt$ $\qquad = w(b \times H \times 1m)$

(Assume 1m length)

$W = sp.wt \times vol$ $\qquad = 21.58 \times 5 \times 10 \times 1$

$W = w \times vol.$ $\qquad \boxed{W = 1079.1 \text{ kN}}$

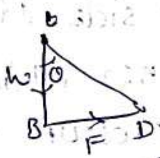
∴ The wt of dam is acting at an centroid

$\frac{b}{2} = \frac{10}{2} = 5m$

iii) Resultant $R = \sqrt{F^2 + W^2}$

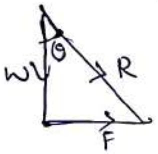
$= \sqrt{(490.5)^2 + (1079.1)^2}$

$= 1185.396$



$\tan\theta = \frac{BD}{OB} = \frac{F}{W}$

$\theta = \tan^{-1}(F/W)$



$\tan\theta = \frac{F}{W}$

$\theta = \tan^{-1}\left(\frac{490.5}{1079.1}\right) = 24°26'$

iv) $F \times \frac{h}{3} = W \times x$

$x = \frac{F}{W} \times \frac{h}{3}$

$490.5 \times \frac{10}{3} = 1051 \times 1079.1$

$x = 1.51$ m

$x = 1.51$ m

v) $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$; $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

$\sigma_{max} = 607.66$ kN/m² (c) ; $\sigma_{min} = -176.33$ kN/m² (

Trapezoidal dam having water force vertical

[Trapezoidal dam having water face vertical]

fig. shows trapezoidal dam having water force vertical. Let h = height of the water

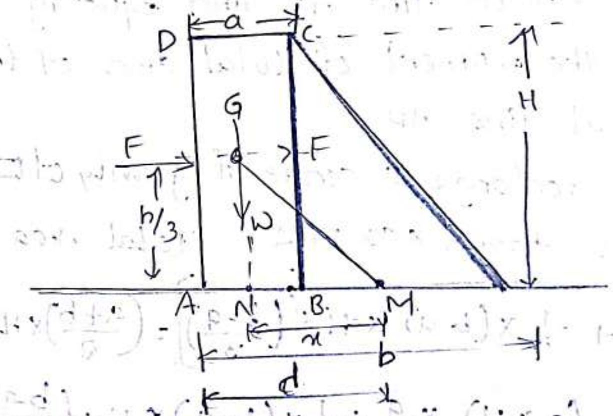
a = top width of dam

G = centre of Gravity

b = bottom width of dam

w_0 = wt. density of water ; w = wt. density masonry

H = Ht. of dam



F = Force exerted by water.

W = wt. of the dam per m length

S-1 :- pressure force due to water :

$F = w \times A \times \bar{h} = \frac{wh^2}{2}$ N

∴ It is also acting at $h/3$ above the base of the dam.

S-2 : To find out the wt. of dam

$= \omega \times vol = \omega \times (A \times L) \quad L = 1m$

$W = \omega \times (A \times L) \qquad A = \left(\frac{a+b}{2}\right) \times H$

$= \omega \times \left(\frac{a+b}{2}\right) \times H \times 1m \qquad$ Assume $L = 1m$

S-3

iii) The distance of centre of gravity of the Trapezoidal section from vertical face AD is obtain by splitting the dam section into a rectangular and triangle, taking the moments of their areas about line AD and equating the same with the moment of total area of trapezoidal section about line AD.

i.e. Area of rectangle × centre of gravity of $\square^{le}$ + Area of triangl × CG of Δ^{le} = Total Area of dam

$$(a \times H) \times \frac{a}{2} + \frac{1}{2} \times (b-a) \times H \times \left[a + \left(\frac{b-a}{3}\right)\right] = \left(\frac{a+b}{2}\right) \times H \times AN$$

$$AN = (a \times H) \times \frac{a}{2} + \frac{1}{2} \times (b-a) \times H \times \left(\frac{b-a}{3}\right)$$

$$AN = \frac{a^2 + ab + b^2}{3 \times (a+b)} \qquad AM = AN + NM$$

$$AM = \frac{a^2 + ab + b^2}{3(a+b)} + x$$

$$d = \frac{a^2 + ab + b^2}{3(a+b)} + \frac{F}{W} \times \frac{h}{3}$$

S-4 : $e = \left(d - \frac{b}{2}\right)$

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right] ; \quad \sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

Q) A masonry retaining wall of trapezoidal section is 10 m high and retains at earth which is level upto the top. The width of the top is 2m. and at the bottom 8 m. and exposed face is vertical. Find the Maximum & minimum Intensities of Normal stress at the base. Take density of the earth 1600 kg/m³ Density of Masonry is equal to 2400 kg/m³. angle of repose of earth = 30°

↳ max inclination at which the particular structure in equilibrium condition

A: Given

height $h = 10m$.

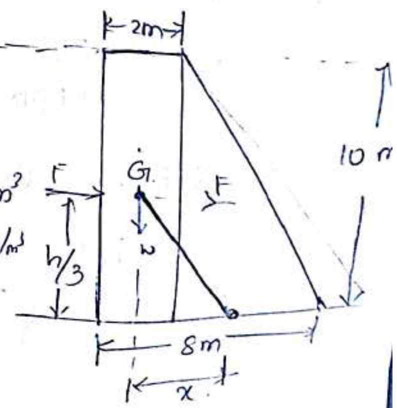
width of top $b = 2m$.

bottom width $= 8m$

density of earth $\rho = 1600\ kg/m^3$

density of Masonry $\rho = 2400\ kg/m^3$

angle of repose $\phi = 30^\circ$



S-1 pressure force due ...

$w = \rho g$

$$F = \frac{wh^2}{2}\left[\frac{1 \mp \sin\phi}{1 + \sin\phi}\right]$$

$$F = \frac{1600 \times 9.81 \times 10^2}{2}\left(\frac{1 - \sin 30}{1 + \sin 30}\right)$$

$F = 261.6$ kN

e +ve → Toe stress max

e −ve → heel stress max

S-2 weight of retaining wall

$W = w \times vol.$

$w = \rho g$

$= 1600 \times 9.81$

$$= w \times \left(\frac{a+b}{2}\right) \times H \times 1m$$

$$= 2400 \times 9.81 \times \left[\frac{2+8}{2}\right] \times 10$$

$= 1.177 \times 10^6$ N $\Rightarrow$ 1177 kN

S-3

We know that

$$AN = \frac{a^2+ab+b^2}{3(a+b)} = \frac{2^2+2(8)+8^2}{3(2+8)} = 2.8$$

$AM = AN + NM \Rightarrow x = \left(\frac{F}{W} \times \frac{h}{3}\right)$

$2.8 + x$

$$= 2.8 + \left[\frac{261.6 \text{ kN}}{1.177 \times 10^3 \text{ kN}} \times \frac{10}{3}\right]$$

$= 2.8 + 0.744$ (m)

d $AM = 3.54$ m

iv) $e = \left(d - \frac{b}{2}\right) \Rightarrow \left(3.54 - \frac{8}{2}\right)$

$= -0.46$

v) $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{1.177 \times 10^3}{8}\left[1 + \frac{6(-0.46)}{8}\right]$

$= 96.36$ kN/m²

$$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right] = \frac{1.177 \times 10^3}{8}\left[1 - \frac{6(-0.46)}{8}\right]$$

$\sigma_{min} = 197.80$ kN/m²

$\sigma_{max} = 197.80$ kN/m² @ Point at A or heel

$\sigma_{min} = 96.36$ kN/m² @ pt. B (or) toe portion

Note: e +ve → stress are max at point B

e −ve → " " " " A

Q) A Masonry trapezoidal Dam 4m high, 7m wide at its top. and 3 m width at its bottom. retains water on its vertical face.. Determine the maximum & Minimum stresses at the base.

i) when reservior is full.

ii) when reservior is empty. Take the weight density of masonry as 19.62 kN/m³.

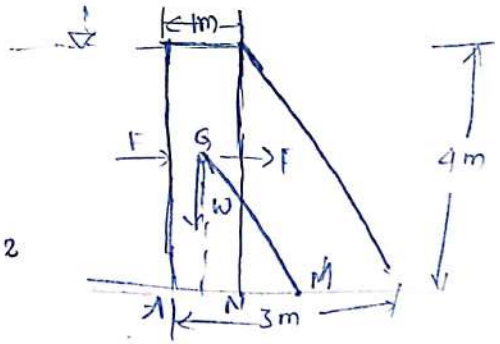
Given

$h = 4$ m.

$a = 1$ m

$b = 3$ m.

wt. density of masonry = 19.62



1)

S-1 pressure force of wat

$F = \frac{wh^2}{2} \Rightarrow \frac{\rho g \times 4^2}{2} \Rightarrow \frac{10^3 \times 9.81 \times 4^2}{2}$

$F = \frac{19.62 \times 4^2}{2} = \cancel{156.96\ kN}$

$= 78.48 \times 10^3$ N.

S-2 wt of dam

$W = \omega \times vol$

$= 19.62 \times \left(\frac{a+b}{2}\right) \times H \times 1\ m$

$= 19.62 \times \left(\frac{1+3}{2}\right) \times 4$

$= 156.96$ KN.

S-3. $AN = \frac{a^2+b^2+ab}{3(a+b)} = \frac{1^2+3^2+1\times3}{3(1+3)} = 1.08\ m$

$NM = x = \frac{F}{W} \times \frac{h}{3} \Rightarrow \frac{78.48}{156.96} \times \frac{4}{3} = 0.66$

$d = AN + AM$

$= 1.08 + 0.66$

$d = 1.74\ m$

iv) $e = \left(d - \frac{b}{2}\right) = \left(1.74 - \frac{3}{2}\right) = 0.25\ m$

v) $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{156.96}{3}\left[1 + \frac{6(0.25)}{3}\right]$

$= 78.48\ kN/m^2$

$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right] = \frac{156.96}{3}\left[1 - \frac{6(0.25)}{3}\right]$

$= 26.16\ kN/m^2$

II) i) F = 0 since reservoir is empty ... only self wt of dam

ii) wt of dam = 156.96

iii) $AN = d = \frac{a^2+b^2+ab}{3(a+b)} = 1.08\ m$

iv) $e = \left[1.08 - \frac{3}{2}\right] = -0.42$

v) $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{156.96}{3}\left[1 + \frac{6(-0.42)}{3}\right]$

$= 8.371\ kN/m^2$

$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right] = \frac{156.96}{3}\left[1 - \frac{6(-0.42)}{3}\right]$

$= 96.26\ kN/m^2$

$\Rightarrow \sigma_{max} = 96.26\ kN/m^2$

$\sigma_{min} = 8.37\ kN/m^2$

Angle of repose :- It is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the resistance of friction only.

→ Angle of repose = angle of friction.

*Stability of the Dam :-

It may fail under the following conditions.

1) By sliding on the soil, It rests. Force due to water 'F' is acting at a distance of H/3.

weight of the dam is acting at a distance of $b/2$.

In order to avoid sliding F_{max} should be greater than F

$$\boxed{F_{max} > F}$$

where, to resist the resultant the resultant force R, the resisting force R^* has been generated at the base of the dam which is have been resolved into two components, the horizontal component is equal to the frictional force and the vertical component is equal to weight of dam

where; $F_{max} = \mu W$

μ = coefficient of friction

2) Condition to prevent the overturning of the dam

⇒ Overturning Moment due to force F about 'M'

about $M \Rightarrow F \times \frac{h}{3} = \omega \times x$

about $B \Rightarrow F \times \frac{h}{3} = W \times (NB)$

$$\omega \times NB > \omega \times x$$

$$\boxed{NB > x}$$

$$\boxed{NB > NM}$$

3) Condition to avoid tension in the masonry dam at its base.

We know that to avoid tension eccentricity

$$e \le \frac{b}{6} \quad \text{(Middle third rule)}$$

also, $e = d - \frac{b}{2}$

$$d - \frac{b}{2} \le \frac{b}{6}$$

$$\boxed{d \le \frac{2}{3} b}$$

where d = Max'm distance b/w A and the resultant force R cuts the base

4) Condition to avoid the excessive compressive stress at the base of the dam.

Max'm stress in Masonry ≤ permissible stresses in Masonry.

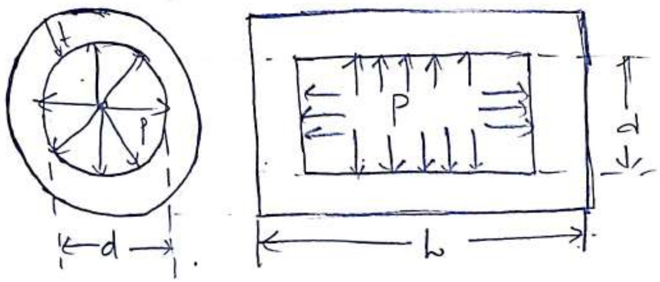
IV Thin & Thick Cylinders

Thin cylinders :-

If the wall thickness of the cylindrical vessel is less than $\frac{1}{15}$ (or) $\frac{1}{20}$ of the internal diameter then the vessel is known as thin cylinder.

Applications of thin cylinders :

1) Boilers
2) compressed air recievers



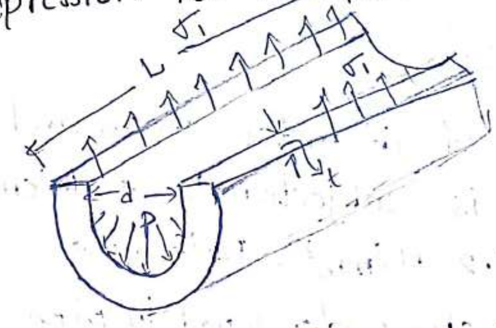
d = Internal diameter
P = Internal pressure of fluid
t = thickness
L = length of the cylinder.

Types of stresses :-

There are two types of stresses which is developed along and perpendicular to the axis & the nature of stress are tensile.

1) circumferential or Hoop stress.
2) Longitudinal stress.

Expression for circumferential or Hoop stress :



Force due to Internal fluid pressure $= P \times A$

$= P \times (d \times L)$ —— ①

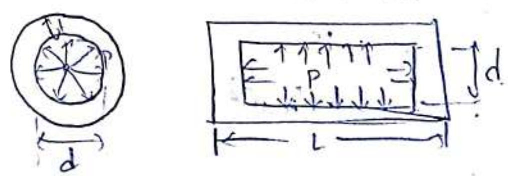
Resisting force $= 2\sigma_1 \times (L \times t)$ —— ②

To maintain equilibrium equale ① = ②

$$P \times (d \times L) = 2\sigma_1 \times (L \times t)$$

$$\sigma_h \text{ (or) } \boxed{\sigma_1 = \frac{Pd}{2t}}$$

Expression for Longitudinal stress.



i) Force due to liquid pressure $= P \times A$

$= P \times \frac{\pi}{4} \times d^2$ ——

ii) Resisting force $= \sigma \times (\pi d \times t)$ —— ②

To maintain equilibrium ① = ②

$$P \times \frac{\pi}{4} \times d^2 = \sigma \times \pi d \times t$$

$$\sigma_L = \sigma_2 = \boxed{\sigma = \frac{Pd}{4t}}$$

Q) A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2. Determine

i) The longitudinal stress developed in pipe

ii) The circumferential stress developed in pipe

$\frac{t}{d} < \frac{1}{20}$ (or) $\Rightarrow \frac{15}{1500} < \frac{1}{20} \Rightarrow 0.01 < 0.05$ ∴ Thin cylinder

Given

$d = 1.5\ m = 1500\ mm$

$t = 1.5\ cm = 0.015\ m = 15\ mm$

$P = 1.2\ N/mm^2$

i) longitudinal stress

$$\sigma = \frac{Pd}{4t}$$

$$= \frac{1.2 \times 1500}{4 \times 15}$$

$$= 30\ N/mm^2$$

ii) hoop stress

$$\sigma_h = \frac{Pd}{2t}$$

$$\sigma_h = \frac{1.2 \times 1500}{2 \times 15}$$

$$\sigma_h = 60\ N/mm^2$$

2) A water main 80 cm dia contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m^3. Find the thickness of the metal required for the water main given the permissible stress as 20 N/mm^2.

$D = 80\ cm = 0.8\ m$

$h = 100\ m$

$\rho = 9810\ N/m^3$

$\sigma = 20\ N/mm^2 = 2 \times 10^{-5}$ ⁴⁄

SSF

$P = \rho g h$ $\quad \frac{N}{mm^2}$

$= 9810 \times \cancel{9.81} \times 100$

$P = 981 \times 10^3\ N/m^2$

$\sigma = \frac{Pd}{2t}$

$P = \rho h$

$= 9810 \times 100$

$P = 981 \times 10^3\ N/$

$\sigma = F$

$$\sigma = \frac{Pd}{2t} \Rightarrow t = \frac{Pd}{2\sigma}$$

$$\frac{\frac{N}{m^2} \times m}{2 \times \frac{N}{m^2}}$$

$= 0.0196\ m = 1.96$ $\quad$ 1.96cm

Maximum Shear stress

$$= \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right)}{2} \Rightarrow \frac{Pd}{8t}$$

Efficiency of the joints;

i) Efficiency of longitudinal joint

$$\eta_L = \frac{Pd}{2t \times \sigma_2}$$

ii) Efficiency of hoop joint

$$\eta_h = \frac{Pd}{4t \times \sigma_1}$$

Q) A boiler is subjected to an internal steel pressure of $2\,N/mm^2$. The thickness of the boiler plate is 2 cm. The permissible tensile stress is $120\,N/mm^2$. Find out the maximum diameter when the efficiency of longitudinal joint is 90% and that of circumference joint is 40%.

$P = 2\,N/mm^2$ $\quad d = ?$

$t = 20\,mm$

$\sigma_T = 120\,N/mm^2$

$$\eta_L = \frac{Pd}{2t \times \sigma_2}$$

$$d = \frac{0.9 \times 2 \times 20 \times 120}{2} \Rightarrow 2160\,mm$$

$$\eta_h = \frac{Pd}{4t \times \sigma_1}$$

$$d = \frac{0.4 \times 4 \times 20 \times 120}{2} \Rightarrow 1920\,mm$$

Strain Developed

We know that

Hoop:
$$e_h = \frac{\sigma_h}{E} - \frac{\mu \sigma_L}{E} \quad (\text{in 2D})$$

$$\boxed{\cancel{e_L = \frac{\sigma_L}{E} - \mu . \frac{\sigma_h}{E}}}$$

Hoop strain

$$\sigma_h = \frac{Pd}{2t} \quad \sigma_L = \frac{Pd}{4t}$$

$$e_h = \frac{Pd}{2tE} - \mu . \frac{Pd}{4tE}$$

Longitudinal strain

$$e_L = \frac{\sigma_L}{E} - \mu . \frac{\sigma_h}{E}$$

$$e_L = \frac{Pd}{4tE} - \mu . \frac{Pd}{2tE}$$

Volumetric strain

1) change in dia

$$e_d = \frac{\text{change in circumference}}{\text{original circumference}}$$

$$= \frac{\text{final} - \text{Initial}}{\text{Initial}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$= \frac{\cancel{\pi d} + \cancel{\pi}\delta d - \cancel{\pi d}}{\cancel{\pi} d}$$

$$e_d = \frac{\delta d}{d}$$ equate $\frac{\delta d}{d}$ to hoop strain

$$e_h = \frac{\delta d}{d}$$

$$\frac{Pd}{2tE} - \mu \cdot \frac{Pd}{4tE} = \frac{\delta d}{d}$$

$$\boxed{\delta d = \frac{Pd^2}{2tE} - \frac{\mu P d^2}{4tE}}$$

Change in length

$$e_L = \frac{\delta L}{L}$$

$$\frac{\delta L}{L} = \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$\delta L = \frac{PdL}{2tE} - \frac{\mu PdL}{4tE}$$

Volumetric strain :-

$$e_v = \frac{\delta V}{V} = \frac{\text{change in vol.}}{\text{Original vol}}$$

$$= \frac{\text{Initial} - \text{final}}{A \times L}$$

$$= \frac{\frac{\pi}{4} \times (d+\delta d)^2 \times (L+\delta L) - \frac{\pi}{4} \times (d^2) \times L}{\frac{\pi}{4} \times d^2 \times L}$$

$$= \frac{\frac{\pi}{4}\left[(d+\delta d)^2 \times (L+\delta L) - (d^2 \times L)\right]}{\frac{\pi}{4} \times d^2 \times L}$$

$$= \frac{(L+\delta L)(d^2 + 2d\delta d + \delta d^2) - d^2 \times L}{d^2 \times L}$$

negle

$$= \frac{\cancel{d^2 l} + d^2\delta l + \cancel{\delta d^2 l} + \cancel{\delta d^2 \times \delta l} + 2d\delta d\, l + \cancel{2d\delta d \delta l} - \cancel{d^2 l}}{d^2 \times l}$$

$$= \frac{d^2\delta l + 2d\delta d\, l}{d^2 \times l}$$

$$= \frac{\cancel{d^2}\delta \cancel{l}}{\cancel{d^2} \times \cancel{l}} + \frac{2\cancel{d}\delta d\, \cancel{l}}{d^{\cancel{2}} \times \cancel{l}}$$

$$= \frac{\delta l}{l} + \frac{2\delta d}{d}$$

$$= e_L + 2e_d$$

$$\boxed{e_v = e_L + 2e_d}$$

$$= \left[\frac{\frac{PdL}{4tE} - \frac{\mu \cdot PdL}{2tE}}{L}\right] + \left[\frac{\frac{Pd^2}{4tE} - \frac{\mu Pd^2}{2 \cdot 4 tE}}{d}\right]2$$

$$= \frac{Pd}{4tE} - \frac{\mu Pd}{2tE} + \frac{Pd}{tE} - \frac{\mu Pd}{2tE}$$

$$= \frac{Pd}{tE}\left[\frac{1}{4}+1\right] - \frac{\mu Pd}{tE}$$

$$= \frac{5}{4}\frac{Pd}{tE} - \frac{\mu Pd}{tE}\frac{8}{4} \Rightarrow \frac{5Pd}{tE} - \frac{\mu Pd}{tE}$$

$$\boxed{e_v = \frac{3Pd}{2tE}\left[1-\frac{\mu}{2}\right]}$$

Q) Calculate i) change in dia ii) change in length iii) change in volume of a thin cylindrical shell of dia 100 cm and 1 cm thick and 5 m long. when subjected to Internal pressure of 3 N/mm², Take. the value of E = 2×10^5 N/mm² ; and poisson's ratio = 0.3

Given.

$d = 100$ cm $= 1000$ mm

$t = 1$ cm $= 10$ mm

$l = 5$ m $= 5000$ mm

$P = 3$ N/mm²

$E = 2\times10^5$ N/mm²

$$\delta d = \frac{Pd^2}{2tE} - \frac{\mu \cdot Pd^2}{4tE}$$

$$\delta d = \frac{3\times1000^2}{2\times10\times2\times10^5} - \frac{0.3\times3\times1000^2}{4\times10\times2\times10^5}$$

$\delta d = 0.6375$ mm.

$$\delta l = \frac{PdL}{4tE} - \frac{\mu PdL}{2tE}$$

$$\delta l = \frac{3\times1000\times5000}{4\times10\times2\times10^5} - \frac{0.3\times3\times1000\times5000}{2\times10\times2\times10^5}$$

$\delta l = 0.75$ mm

$\delta V = e_v \times V$

$$\delta V = \frac{\pi}{4}(d+\delta d)^2(l+\delta l) - \frac{\pi}{4}d^2 l$$

$$\boxed{\delta_V = \frac{Pd^2}{2tE}\left[\frac{5}{2} - 2\mu\right]}$$

$$\frac{\delta V}{V} = \left[\frac{5}{4}\frac{Pd}{tE} - \frac{\mu Pd}{tE}\right]$$

$$= \frac{5}{4}\times\frac{3\times1000}{10\times2\times10^5} - \frac{0.3\times3\times1000}{10\times2\times10^5}$$

$= 0.00142$

$$\frac{\delta V}{V} = 0.00142 \Rightarrow \delta V = \frac{\pi}{4}\times(1000)^2\times5000\times0.00142$$

$= 5.576\times10^6$ mm³

Q)- A cylindrical vessel whose ends are closed by means of rigid flange plate is made of steel 3 mm thick the length and internal dia of vessel are 50 cm. and 25 cm respectively determine longitudinal and hoop stress in the cylindrical shell due to internal fluid pressure of 3 N/mm^2. also calculate the increase in length, dia & volume of vessel. Take $E = 2\times10^5$ N/mm^2 and poisson's ratio $= 0.3$

$t = 3$ mm

$l = 50$ cm $= 500$ mm

$d = 25$ cm $= 250$ mm

$D = d - 2t = 250 - 6 = 244$ m

$P = 3$ N/mm^2 ; $E = 2\times10^5$ N/mm^2

Longi $\sigma = 62.2$ N/m

hoo $\sigma = 125$ N/m

$\delta l = 0.06$ mm

$\delta d = 0.13$ mm

$\delta v = 29.1456\times10^3$ mm^3

$$\sigma_h = \frac{Pd}{2t} \Rightarrow \frac{3\times250}{2\times3} = 125 \text{ N/mm}^2$$

$$\sigma_L = \frac{Pd}{4t} = \frac{3\times250}{4\times3} = 62.5 \text{ N/mm}^2$$

$$\delta l = \frac{PdL}{2tE} - \frac{\mu PdL}{4tE}$$

$$= \frac{3\times250\times500}{2\times3\times2\times10^5} - \frac{0.3\times3\times250\times506}{4\times3\times2\times10^5} = 0.06 \text{ mm}$$

Thin Spherical shells :

The force (P) which has a tendency to split the shell the fluid pressure $F = P\times\frac{\pi}{4}\times d^2$

Area resisting this force $= \sigma\times\pi d\times t$

for equilibrium, $P\times\frac{\pi}{4}\times d^2 = \sigma\times\pi\times d\times t \Rightarrow \sigma = \frac{P\times d}{4\times t}$

$$\boxed{\text{Hoop stress} = \text{Longitudinal stress} = \frac{Pd}{4t}}$$

strain :

$$e = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$\sigma_1 = \sigma_2 = \frac{Pd}{4t}$$

$$e = \frac{Pd}{4tE} - \frac{\mu Pd}{4tE}$$

Strain (e) $= \frac{Pd}{4tE}(1-\mu)$

Change in dia :- (δd)

Lateral strain $e = \frac{\delta d}{d} = \frac{Pd}{4tE}(1-\mu)$

$$\frac{\delta d}{d} = \frac{Pd}{4tE}(1-\mu)$$

$$\boxed{\delta d = \frac{Pd^2}{4tE}(1-\mu)}$$

Change in Volume (δv) :-

$V = \frac{4}{3}\pi r^3$ $\boxed{d = 2r}$

$$V = \frac{4}{3}\times\pi\left(\frac{d}{2}\right)^3$$

Q) $V = \frac{4}{3} \times \pi \left(\frac{d^3}{8_2}\right)$ $\delta V = \frac{dV}{dd}$

$V = \frac{\pi d^3}{6}$ $\frac{d}{dd}\left[\frac{\pi d^3}{6}\right] = \frac{\pi}{6}(3d^2)$

$\frac{dv}{dd}\delta v = \frac{\pi}{6_2}[3d^2]\delta d$

$\delta V = \frac{\pi d^2}{2} \times \delta d$

$\delta v = \frac{\pi d^2}{2}\left[\frac{Pd^2}{4tE}(1-\mu)\right]$

$\delta v = \frac{\pi d^2}{2} \cdot \delta d$

$e = \frac{\delta v}{v}$

$e = \dfrac{\frac{\pi d^3}{2} \cdot \delta d}{\frac{\pi d^3}{6_3}}$

$\frac{\delta V}{V} = \frac{3 \cdot \delta d}{d}$

$\boxed{e_v = 3 e_d}$

Q) A vessel in the shape of a spherical shell 1.2 m internal dia 12 mm shell thickness subjected to pressure of 1.6 N/mm². Determine the stress induced in material of vessel

Given; $d = 1.2\,m = 1200\,mm$

$t = 12\,mm$

$D = d + 2t = 1200 + 2 \times 12$

$= 1224$

$P = 1.6\ N/mm^2$

$\sigma = \frac{Pd}{4t} = \frac{1.6 \times 1224}{4 \times 12} = 40.8\ N/mm^2$

Q) A closed cylindrical vessel steel plate with 4 mm thick. plain ends & carries a fluid under a pres of 3 N/mm² the dia of cylinder is 25 cm & length 75 cm. calculate the longitudinal & hoop stress in the cylinder wall and determine the change in dia, length & volume of cylinder, Take $E = 2.1 \times 10^5\ N/mm^2$ & $\mu = 0.286$

$t = 4\,mm$, $p = 3\ N/mm^2$, $d = 25\,cm = 250\,mm$, $l = 75\,cm = 750\,mm$

$E = 2.1 \times 10^5\ N/mm^2$, $\mu = 0.286$

$\sigma_c = \frac{Pd}{2t} = \frac{3 \times 250}{2 \times 4} = 93.75\ N/mm^2$

$\sigma_h = \frac{Pd}{4t} = \frac{3 \times 250}{4 \times 4} = 46.875\ N/mm^2$

Torgot

$$e = \frac{\sigma_c}{E} - \mu \cdot \frac{\sigma_h}{E}$$

$$e = \frac{Pd}{2tE} - \mu \cdot \frac{Pd}{4tE}$$

$$\frac{\delta d}{d} = \frac{Pd}{2tE}\left[1 - \mu \cdot \frac{1}{2}\right]$$

$$\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$\delta d = \frac{3 \times 250^2}{2 \times 4 \times 2.1 \times 10^5}\left[1 - \frac{0.286}{2}\right]$$

$$\delta d = 0.095 \text{ mm}$$

Change in length (δl)

$$e = \frac{Pd}{4tE} - \mu \cdot \frac{Pd}{2tE}$$

$$e = \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\delta l = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right] \times L$$

$$\delta l = \frac{3 \times 250}{2 \times 4 \times 2.1 \times 10^5}\left[\frac{1}{2} - 0.286\right] \times 750$$

$$\delta l = 0.0716 \text{ mm}$$

Change in Volume:

$$\delta v = \frac{Pd^2}{2tE}\left[\frac{5}{2} - 2\mu\right]$$

$$\delta v = \frac{3 \times 250^2}{2 \times 4 \times 2.1 \times 10^5}\left[\frac{5}{2} - 2 \times 0.286\right]$$

$$\delta v = 0.215 \text{ mm}^3$$

Thick cylinders:

→ In thin cylinders since the thickness is very small, therefore radial stresses are neglected

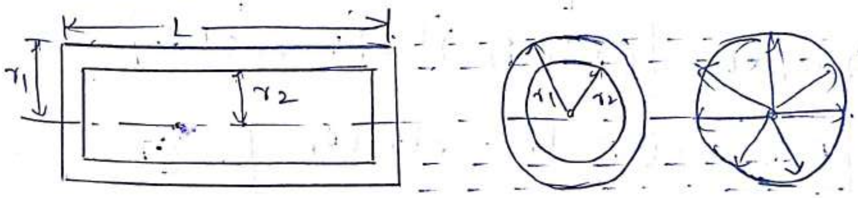
→ circumferential stresses are acting along circumferential direction

→ Longitudinal stresses are acting along the longitudinal axis.

→ Radial stress acts along the thickness in thick cylinders.

→ Let r_1 be the radius from centre to outer surface of cylinder.

→ Let r_2 be the radius acting from center to inner surface of cylinder.



→ Let 'L' be the length, consider a small elementy ring in the thickness of cylinder.

→ stress acting on the element ring are σ_r.

→ 'r' radius be the centre to element ring

→ σ_r be the radial stress along on the cylinder

→ σ_r stress acting on radial.

→ Let σ_h be the hoop stress induced in ring.

→ P represents the internal fluid pressure

→ δr be the thickness of element ring

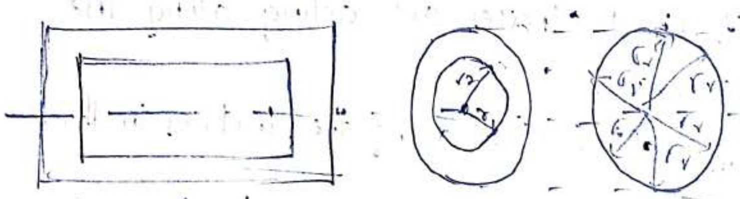
→ ~~pre~~ Derivation

i) Bursting force

$F = P \times A$

Pressure at inner surface > pressure @ outer surface

$= [P \times A]_{inner} - [P \times A]_{outer}$



Area of elementary ring

$A = 2r \times L$

$= (\sigma_r \times A)_{in} - [(\sigma_r + d\sigma_r) \times A]$

$= \sigma_r \times (\sigma_r \times L) - [(\sigma_r + d\sigma_r) \times [2(r + dr) + l]]$

$= 2r\sigma_r \times L - [(\sigma_r + d\sigma_r)] \times [2lr + 2ldr]$

$$\boxed{F = -2l\,\sigma_r\,dr - 2lr\,d\sigma_r} \quad \text{— (1)}$$

Resisting force.

$F = P \times A$

$$\boxed{\sigma_h \times (2dr \times l)} \quad \text{— (2)}$$

For equilibrium (1) = (2)

$-2l\sigma_r dr - 2lr\,d\sigma_r = \sigma_h\,(2dr \times l)$

$-\cancel{2l}\,[\sigma_r dr + r\,d\sigma_r] = \cancel{2l}\,(\sigma_h dr)$

$-[\sigma_r dr + r\,d\sigma_r] = \sigma_h dr$

$\sigma_h = -\left[\frac{\sigma_r dr}{dr} + \frac{r\,d\sigma_r}{dr}\right]$

$$\boxed{\sigma_h = -\sigma_r - \frac{r\,d\sigma_r}{dr}} \quad \text{— (3)}$$

Note: σ_r is always compressive so σ_r = '−ve'

(ii) Strain

$e_L = \frac{\sigma_L}{E} - \frac{\mu \cdot \sigma_h}{E} - \frac{\mu(-\sigma_r)}{E}$

$e_L = \frac{\sigma_L}{E} - \frac{\mu \cdot \sigma_h}{E} + \frac{\mu \cdot \sigma_r}{E}$

$e_L = \frac{\sigma_L}{E} - \frac{\mu}{E}(\sigma_h - \sigma_r)$

$e_L - \frac{\sigma_L}{E} = -\frac{\mu}{E}(\sigma_h - \sigma_r)$

$\left(e_L - \frac{\sigma_L}{E}\right)\frac{E}{\mu} = [\sigma_r - \sigma_h]$

From the Assumptions of Layer's strain is const

→ The longitudinal strain at any point in section is const. and is independent of radius (from assumptions)

→ plain section before strain remains plane after strain

→ ~~As~~ the longitudinal strain is const & according to hook's law

$\underbrace{\left[e_L - \frac{\sigma_L}{E}\right]\frac{E}{\mu}}_{\text{constant}} = (\sigma_r - \sigma_h)$

$= \sigma_r - \sigma_h$

$-2a = \sigma_r - \sigma_h$

$$\boxed{\sigma_h - \sigma_r = 2a} \longrightarrow (4)$$

$\sigma_h = \sigma_r + 2a$

Equating (3) & (4)

$$-\sigma_r - \frac{r\,d\sigma_r}{dr} = \sigma_r + 2a$$

$$-\frac{r\,d\sigma_r}{dr} = 2\sigma_r + 2a$$

$$-\frac{r\,d\sigma_r}{dr} = 2(\sigma_r + a)$$

$$\frac{d\sigma_r}{(\sigma_r + a)} = \frac{-2dr}{r}$$

I.O.B.S

$$\log(\sigma_r + a) = -2\log r + \underbrace{\log b}_{\text{Integration const.}}$$

$$\log_e(\sigma_r + a) = \log r^{-2} + \log b$$

$$\log(\sigma_r + a) = \log b r^{-2}$$

$$\sigma_r + a = \frac{b}{r^2}$$

$$\boxed{\sigma_r = \frac{b}{r^2} - a} \longrightarrow (5)$$

C Put σ_r in eq. (4)

$$\sigma_h - \sigma_r = 2a$$

$$\sigma_h - \left[\frac{b}{r^2} - a\right] = 2a$$

$$\sigma_h = 2a + \frac{b}{r^2} - a$$

$$\boxed{\sigma_h = \frac{b}{r^2} + a} \longleftarrow (6)$$

$\therefore$ eqn's (5) & (6) are called as lame's eqn :

a & b are called lame's const.

Q) Determine the maximum hoop stress and minimum hoop stress across the Section of the pipe of 400 mm internal dia and 100 mm thick when the pipe contains a fluid at pressure of 8 N/mm². Also sketch the radial pressure and hoop stress distribution across the Section.

Given,

Internal dia $d = 400$ mm

Thickness $t = 100$ mm

fluid pressure $p = 8$ N/mm².

$D = d + 2t = 400 + 2(100) = 600$

$\frac{t}{d} \not> \frac{1}{20} \Rightarrow 0.25 > 0.05$ ∴ Thick cylinder

r_i Internal radius $= 200$ mm

r_e External radius $= 300$ mm

W.K.T

$\sigma_r = \frac{b}{r^2} - a$

at $r = r_i \Rightarrow \sigma_r = P = 8$ N/mm²

$8 = \frac{b}{200^2} - a \Rightarrow$ ①

at $r = r_e \Rightarrow \sigma_r = 0$

$0 = \frac{b}{300^2} - a \Rightarrow$ ②

∴ from ② $a = \frac{b}{(300)^2}$

in ① $8 = \frac{b}{(200)^2} - \frac{b}{(300)^2}$

$= \frac{b}{(100)^2}\left[\frac{1}{4} - \frac{1}{9}\right]$

$8 = \frac{5}{36 \times 10^4} b$

$b = 576 \times 10^3$

$a = \frac{b}{(300)^2} = \frac{576 \times 10^3}{(300)^2} = 6.4$

$\sigma_h \Rightarrow$ @ $r = r_e = (\sigma_h)_{min}$

$\frac{b}{r_e^2} + a = (\sigma_h)_{min}$

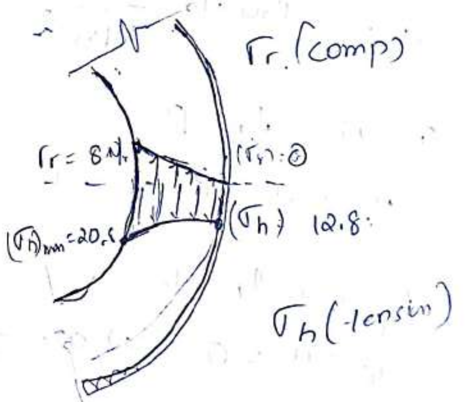
$\frac{576 \times 10^3}{(300)^2} + 6.4 = (\sigma_h)_{min}$

12.8 N/mm² $= (\sigma_h)_{min}$

$(\sigma_h)_{max} = \frac{b}{r^2} + a$ @ $r = r_i$

$(\sigma_h)_{max} = \frac{576 \times 10^3}{(200)^2} + 6.4$

$(\sigma_h)_{max} = 20.8$ N/mm²



Q) A thick cylinder of inner surface radius 150 mm and outer surface radius 210 mm is subjected to internal fluid pressure (P) such that the max'm hoop stress developed in cylinder is 154.16 MPa. Draw the hoop stress and radial stress along the thickness of cylinder. E = 200 GPa. μ = 0.3. What is the circumferential strain in cylinder at outer surface

Given

Inner radius r_i = 150 mm

Outer radius r_e = 210 mm

max. hoop stress = 154.16 MPa

E = 200 GPa

μ = 0.3

$$(\sigma_h)_{max} = \frac{b}{r^2} + a \quad @ \; r = r_i$$

$$154.16 = \frac{b}{(150)^2} + a$$

$$a = 154.16 - \frac{b}{(150)^2}$$

$\frac{\sigma_{hmin}}{E} - \frac{\mu(\sigma_r)}{E} = \epsilon_h$

σ_{hmin} $r = r_e$ $\frac{b}{r_e^2} + a = \sigma_{hmin}$

$b = 2.296 \times 10^6$

$a = 52.063$

C

i) @ $r = r_i$, $\sigma_r = P$

$$\frac{b}{r_i^2} - a = P \quad \text{—— (1)}$$

ii) @ $r = r_e$, $\sigma_r = 0$

$$\frac{b}{r_e^2} - a = 0 \Rightarrow a = \frac{b}{r_e^2} \quad \text{—— (2)}$$

(2) in (1)

$$\frac{b}{r_i^2} - \frac{b}{r_e^2} = P$$

$$b\left[\frac{r_e^2 - r_i^2}{r_i^2 \cdot r_e^2}\right] = P$$

$$\left\{ b\left[\frac{210^2 - 150^2}{210^2 \cdot 150^2}\right] = P \right.$$

$$\left. b = P\left[45937.5\right] \right\}$$

$$\boxed{b = P\left(\frac{r_i^2 \cdot r_e^2}{r_e^2 - r_i^2}\right)}$$

50×

$$a = \frac{1}{r_e^2}\left[\frac{P \cdot r_i^2 \cdot r_e^2}{r_e^2 - r_i^2}\right]$$

$$\boxed{a = \frac{P r_i^2}{r_e^2 - r_i^2}}$$

$(\sigma_h)_{max}$ @ $r = r_i$

$$\frac{b}{r_i^2} + a = (\sigma_h)_{max} = 154.16$$

$$\frac{P r_e^2}{r_e^2 - r_i^2} + \frac{P r_i^2}{r_e^2 - r_i^2} = 154.16$$

$$P\left[\frac{r_e^2 + r_i^2}{r_e^2 - r_i^2}\right] = 154.16$$

$$P\left(\frac{210^2 + 150^2}{210^2 - 150^2}\right) = 154.16$$

$$P(3.082) = 154.16$$

$$P = 50.00 \; N/mm^2$$

$b = 2.296 \times 10^{6} \Rightarrow P\left[\frac{\ldots}{r_i^2 - r_e^2}\right]$

$a = 52.063 \Rightarrow P\left[\frac{r_i^2}{r_e^2 - r_i^2}\right]$

@ at $r = r_i$ $(\sigma_h) = max$

@ $r = r_0$ $(\sigma_r) = 0$, $(\sigma_h) = min$

$$(\sigma_h)_{min} = \frac{b}{r_e^2} + a$$

$$= \frac{2.296 \times 10^{2}}{210^2} + 52.063$$

$$(\sigma_h)_{min} = 104.126 \ N/mm^2$$

Circumferential strain

$$E_h = \frac{(\sigma_h)_{min}}{E} + \frac{\mu (\sigma_r)}{E} \quad \text{compress.} \to 0$$

$$E_h = \frac{104.126}{2 \times 10^5}$$

$$E_h = 5.201 \times 10^{-4}$$

Strengthening of cylinders

In case of cylinders which has to carry higher fluid pressure certain methods are used to reduce the fluid pressure. There are two methods

i) winding wire around the pipe or cylinder

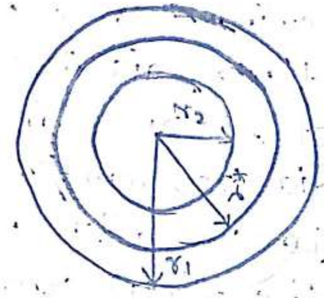
ii) Compound thick cylinders

i) winding wire around the cylinder

This method is applicable for thin cylinders in which a wire is ~~winc~~ wounded around the cylinder along the longitudinal section or axis.

ii) compounding of cylinder

This method is used in thick cylinders to resist the internal fluid pressure. In this method a compound cylinder is formed by inserting (or) shrinking Bigger cylinder over a Smaller cylinder.



Let r_1 be the outer radius

r_2 be the Inner radius

r^* be the Junction radius

$\sigma_r^* =$ Juction pressure.

$\sigma_r =$ Radial stress

$\sigma_h =$ Hoop / circumferential stress

Case (i) :- Empty condition

Outer cylinder.

(i) At $r = r_1$, $\sigma_r = 0$

ii) At $r = r^*$, $\sigma_r = \sigma_r^*$

Inner cylinder

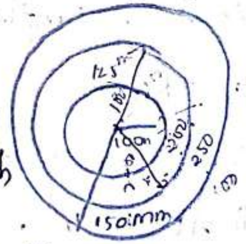
i) At $r = r_2$, $\sigma_r = 0$ [$\because$ no fluid pressure

ii) At $r = r^*$, $\sigma_r = \sigma_r^*$

Case ii) Internal fluid is present -
For outer cylinder
i) At $r = r_1$, $\sigma_r = 0$
For Inner cylinder
ii) At $r = r_2$, $\sigma_r = P$ (internal fluid pressure)
iii) At $r = r^*$, $\sigma_r = \sigma_r^*$

Q) A compound cylinder is made by shrinking a cylinder of external dia 300 mm and internal dia 250 mm over another cylinder of external dia 250 mm and Internal dia 200 mm. The radial pressure at the junction after shrinking is 8 N/mm². Find the final stress set up across the section when the compound cylinder is subjected to Internal fluid pressure of 84.5 N/mm²

Given



{Internal radius = $\frac{250}{2} = 125$ mm}

C

① Outer cylinder
External dia = 300 mm, $r_1 = 150$ mm
Internal dia = 250 mm,
} $r^* = 125$ mm
② Inner cylinder
External dia = 250 mm
Internal dia = 200 mm = $r_2 = 100$ mm
radial pressure at junction after shrinking = 8 N/mm²

Internal fluid pressure = 84.5 N/mm²

Case i) Empty condition

for outer cylinder

i) At $r = r_1$, $\sigma_r = 0$, $r_1 = 150$ mm

$$\sigma_r = \frac{b}{r_1^2} - a$$

$$0 = \frac{b}{(150)^2} - a \Rightarrow a = \frac{b}{(150)^2} \quad \text{— ①}$$

(ii) at $r = r^*$, $\sigma_r = \sigma_r^*$

$$\sigma_r \Rightarrow \frac{b}{(r^*)^2} - a = 8 \text{ N/mm}^2$$

$$= \frac{b}{(125)^2} - a = 8$$

$$= \frac{b}{15.625\times10^3} - \frac{b}{22.5\times10^3} = 8$$

$$\left\{ \frac{b}{1.96\times10^{-5}} = 8 \right\} \quad b\,[1.76\times10^{-4}] = 8$$

$b = 41025.6$

$a = 18.23$

At $r = r_1$, $\sigma_r = 0$, $\sigma_h = 0$

$r = 150$ mm, $\sigma_r = 0$, $\sigma_h = \frac{b}{r^2} + a = 36.46$ N/mm²

$r = 125$ mm, $\sigma_r = 8$ N/mm², $\sigma_h = \frac{b}{125^2} + a = 44.48$

$r = 100$ mm, $\sigma_h = 59.25$

Inner cylinder

At $r = r_2$, $\sigma_r = 0$

$$r_2 = 100 \Rightarrow \frac{b}{100^2} - a = 0 \quad \text{— ①}$$

$a = \frac{b}{(100)^2}$

ii) at $r = r^{*} = 125$ mm, $\sigma_r = \sigma_r^{*} \Rightarrow 8$

$$8 = \frac{b}{(125)^2} - a$$

$$8 = \frac{b}{(125)^2} - \frac{b}{(100)^2}$$

$$8 = b\left(-3.6\times10^{-5}\right)$$

$$b = -2.222\times10^{5} \Rightarrow -222.2\times10^{3}$$

$$a = -22.222$$

@ $r = 100$ mm, $\sigma_r = 0$, $\sigma_h = \frac{b}{100^2} + a = -44.44$ N/mm²

@ $r = 125$ mm, $\sigma_r = 8$, $\sigma_h = \frac{b}{(125)^2} + a = -36.42$ N/mm²

Case ii)

For outer

@ $r = r_1$, $\sigma_r = 0 \Rightarrow 150 \Rightarrow \frac{b}{(150)^2} - a = 0$

$$a = \frac{b}{150^2}$$

For inner

@ $r = r_1$, $\sigma_r = p \Rightarrow 100$ mm $\Rightarrow \frac{b}{(100)^2} - a = 84.5$

$$\frac{b}{100^2} - \frac{b}{150^2} = 84.5$$

$$b\left(5.55\times10^{-5}\right) = 84.5$$

$$b = 1.522522\times10^{6}$$

$$a = 67.66$$

(Hoop stress) Empty, fluid is ...

$$\sigma_h = \frac{b}{r_1^2} + a \Rightarrow \frac{1.525\times10^6}{(150)^2} + 67.66 = (\sigma_h)_{min}$$

$$\sigma_{150} = 135.21 \text{ N/mm}^2$$

$$\sigma_{100} = \frac{1.52\times10^6}{100^2} + 67.66 \Rightarrow 219.66 \text{ N/mm}^2$$

$$\sigma_{125} = \frac{1.525\times10^6}{125^2} + 67.66 \Rightarrow 165.26 \text{ N/mm}^2$$

The resultant stresses will be the algebraic sum of the initial stresses due to shrinking and those due to internal fluid pressure.

For outer Cylinder

$$\sigma_{150} = (\sigma_{150})_{emp} + (\sigma_{150})_{\text{fluid present}} = 36.46 + 135.2 \Rightarrow 171.6 \text{ N/mm}^2$$

$$\sigma_{125} = (\sigma_{125})_{emp} + (\sigma_{125})_{F.P} = 44.36 + 164.94 \Rightarrow 209.3$$

For Inner Cylinder

$$\sigma_{125} = (\sigma_{125})_{emp} + (\sigma_{125})_{FP} = -36.44 + 164.94 = 128.5 \text{ N/mm}^2$$

$$\sigma_{100} = (\sigma_{100})_{emp} + (\sigma_{100})_{FP} = -44.44 + 219.7 \Rightarrow 175.26 \text{ N/mm}^2$$

Q) A steel cylinder of 300 mm external dia. is to be shrunk to another steel cylinder of 150 mm internal diameter. After shrinking, the dia at the junction is 250 mm and radial pressure at the common junction is

28 N/mm^2 . Find the original difference in radii at the junction. Take $E = 2 \times 10^5 \ N/mm^2$.

(1) External dia $D = 300$ mm $r_1 = 150$ mm

Inner dia $d = 150$ mm , $r_2 = 75$ mm

(2) External-Data = 150 mm

Junction dia = 250 mm

$r^* = 125$ mm

$p = 0$

radial pressure $\sigma_r^* = 28 \ N/mm^2$

$E = 2 \times 10^5 \ N/mm^2$

*) Empty condition

for outer cylinder

i) @ $r = r_1$, $\sigma_r = 0$ $\Rightarrow \sigma_r = \frac{b}{r^2} - a$

$$0 = \frac{b}{150^2} - a \Rightarrow a = \frac{b}{150^2}$$

ii) @ $r = r^*$, $\sigma_r = \sigma_r^*$ $\Rightarrow \sigma_r^* = \frac{b}{r_*^2} - a$

$$28 = \frac{b}{125^2} - \frac{b}{150^2}$$

$$28 = b\left[1.95 \times 10^{-5}\right]$$

$$b_1 = 1.43589 \times 10^{6}$$

$$a_1 = \frac{b}{150^2} = 63.77$$

for inner cylinder

i) @ $r = r_2$, $\sigma_r = 0 \Rightarrow 0 = \frac{b}{75^2} - a$

$$a = \frac{b}{75^2}$$

ii) @ $r = r^*$, $\sigma_r = \sigma_r^* \Rightarrow 28 = \frac{b}{125^2} - a$

$$28 = \frac{b}{125^2} - \frac{b}{75^2}$$

$$28 = b\left[-1.137 \times 10^{-4}\right]$$

$$b_2 = -246.26 \times 10^3$$

$$a_2 = -43.77$$

diff. in radii at the junction $= \frac{2r^*(a_1 - a_2)}{E}$

$$= \frac{2 \times 125 (63.77 + 43.77)}{2 \times 10^5}$$

$$= 0.13 \text{ mm}$$

Thick Spherical shells :

i) $\sigma_h = \frac{b}{r^3} + a$

ii) $\sigma_r = \frac{2b}{r^3} - a$

Q) A thick spherical shell of 200 mm internal dia is subjected to an internal fluid pressure of 7 N/mm^2. If the permissible tensile stress in the shell material is 8 N/mm^2, Find the thickness of the shell

Internal dia = 200 mm , $r_2 = 100$ mm

Internal fluid pressure $p = 7$ N/mm^2

Permissible Tensile stress $\sigma_h = 8$ N/mm^2

1) @ $r = r_2$, $\sigma_r = p \Rightarrow$

$$\left\{\begin{array}{l} 7 = \dfrac{b}{r_2^2} - a \\ a = \cancel{7} \dfrac{b}{100^2} - 7 \end{array}\right\}$$

$$\frac{2b}{100^3} - a = 7$$

$$a = \frac{2b}{100^3} - 7$$

ii) @ $r = r_2$, $\sigma_h = 8$ N/mm^2 $\Rightarrow \dfrac{b}{(100)^3} + a = 8$ N/mm^2

$$8 = \frac{b}{100^3} + \frac{2b}{100^3} - 7$$

$$\left\{\begin{array}{l} 8 = \dfrac{b}{100^3} + \left(7 - \dfrac{2b}{100^3}\right) \\ 8 = \dfrac{b}{100^3} - \dfrac{2b}{100^3} + 7 \\ 8 = b(2) + 7 \end{array}\right\}$$

$$8 = \frac{3b}{100^3} - 7$$

$$b = 5 \times 10^6$$

$$a = 3$$

iii) at $r = r_1$, $\sigma_r = 0$

$$\frac{2b}{(r_1)^3} - a = 0$$

$$\frac{2 \times 5 \times 10^6}{(r_1)^3} = 3$$

$$r_1 = 149.38$$

$$t = r_1 - r_2$$

$$= 149.38 - 100$$

$$t = 49.38 \text{ mm}$$

Unsymmetrical Bending

Unsymmetrical bending ;

If the beam is not bending wrt symmetrical axis then it is called unsymmetrical bending

(or)

when the plane of symmetry is not coinciding with plane of loading then it is called unsymmetrical bending.

→ This occurs in two cases

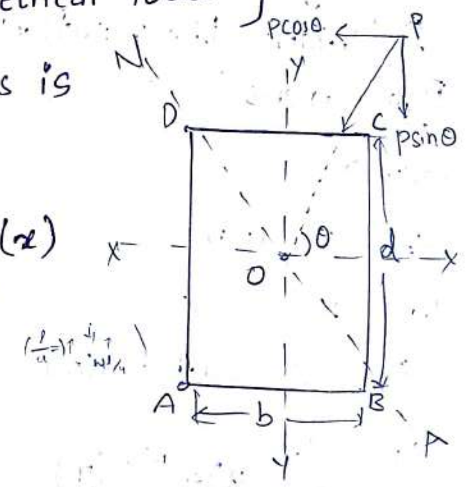
i) when loading is not symmetrical

ii) when cross section is unsymmetrical

Case (i)

In case of unsymmetrical loading

The resultant stress is given by



$$\sigma_R = \frac{M_x}{I_{xx}}(y) + \frac{M_y}{I_{yy}}(x)$$

where,

$$M_x = \frac{P\cos\theta \times l}{4} ;$$

$$M_y = \frac{P\sin\theta \times l}{4}$$

$$I_{xx} = \frac{bd^3}{12} , \quad I_{yy} = \frac{db^3}{12}$$

$x, y \rightarrow$ co-ordinates at given point

step-2 : Location or position of Neutral axis

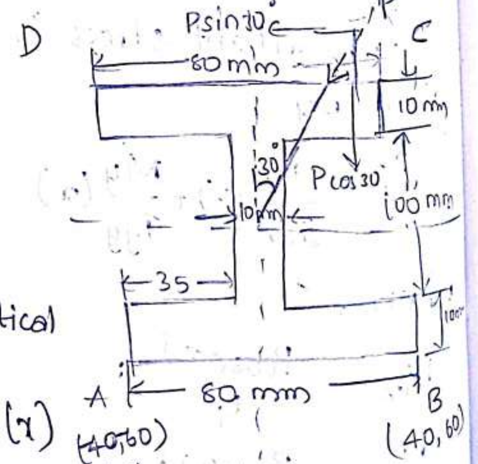
At Neutral Axis : , $\sigma_R = 0$

$$\frac{M_x}{I_{xx}}(y) + \frac{M_y}{I_{yy}}(x) = 0$$

$$\frac{M_x}{I_{xx}}(y) = -\frac{M_y}{I_{yy}}(x)$$

$$\tan\phi = \frac{y}{x} = -\left[\frac{M_y}{I_{yy}} \times \frac{I_{xx}}{M_x}\right]$$

Q1 An I-section shown in simply supported beam of span 4 m, from the central point loading 500 kN at 30° with vertical. Determine the resultant stress at all the four corners



Given

Span $l = 4$ m

load $P = 500$ kN

$\theta = 30°$ with vertical

$$\sigma_R = \frac{M_x}{I_{xx}}(y) + \frac{M_y}{I_{yy}}(x)$$

$$M_x = P\sin\theta \times \frac{l}{4} \Rightarrow 500 \times \sin 30 \times \frac{4}{4}$$

$M_x = 226.99 ~~kN·m~~$

250×10^3 N·m

$$M_y = \frac{P\cos 30° \times l}{4} = \frac{500 \times \cos 30° \times 4}{4}$$

$= ~~445.50~~ \; 433.01 \times 10^3$ N·m

$$I_{xx} = \frac{80 \times 120^3}{12} - 2\left[\frac{35 \times 100^3}{12}\right]$$

$= 5.68 \times 10^6$ mm^4

$$I_{yy} = \frac{Db^3}{12} - 2\left(\frac{db^3}{12}\right)$$

$$= \frac{120 \times 80^3}{12} - 2\left(\frac{100 \times 35^3}{12}\right)$$

$I_{yy} = 4.40 \times 10^6$ mm^4

$I_{yy} = I_1 + I_2 + I_3$

$= 2[I_1] + I_2$

$$= 2\left(\frac{d_1 b_1^3}{12}\right) + \frac{d_2 b_2^3}{12} = 2\left(\frac{10 \times 80^3}{12}\right) + \frac{100 \times 10^3}{12}$$

$= 8.65 \times 10^5$ mm

Step 2)
@ pt. A.

$$\sigma_R)_A = \frac{M_x}{I_{xx}}(-y) + \frac{M_y}{I_{yy}}(-x)$$

$$= \frac{250 \times 10^3 \times 10^3}{5.68 \times 10^6}(-60) + \frac{433.01 \times 10^3 \times 10^3}{8.65 \times 10^5}(-$$

$= \{-4.6 \times 10^6\} - 22.75 \times 10^3$ N/mm^2

$$\sigma_R|_B = \frac{M_x}{I_{xx}}(-60) + \frac{M_y}{I_{yy}}(+40)$$

$= +17.38 \times 10^3$ N/mm^2

$$\sigma_R|_C = \frac{M_x}{I_{xx}}(+40) + \frac{M_y}{I_{yy}}(60)$$

$$= 22.75 \times 10^3 \ N/mm^2$$

$$\sigma_R|_D = \frac{M_x}{I_{xx}}(+60) + \frac{M_y}{I_{yy}}(40)$$

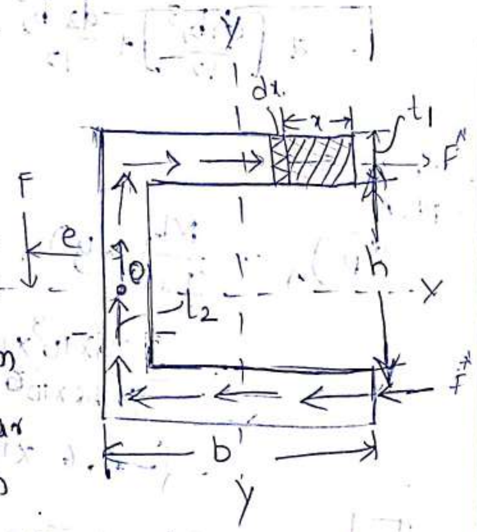
$$= 17.38 \times 10^3 \ N/mm^2$$

Shear centre :-

Shear centre is defined as a point (inside or outside) the section through which applied shear force produces no torsion or twist in the member.

Channel section

Consider a channel section symmetrical about x-axis where



F = shear force applied in the section

F^* = produced shear force in the section

t_1 = thickness of flanges

t_2 = thickness of web

b = length of flange

h = distance b/w line of action of F^*

We know that, the shear stress at any point given in the section is, $\tau = \frac{F A \bar{y}}{I_{xx} b_f}$

where

A = Area of shaded portion $= x \times t_1$

$$\therefore \tau = \frac{F \times (x \times \cancel{t_1}) \times \frac{h}{2}}{I_{xx} \times \cancel{t_1}}$$

$$\tau = \frac{F x h}{2 I_{xx}}$$

Step 2 :- Shear force produced in flanges

$$F^* = \tau \times dA$$

$$F^* = \int \tau \times dA \qquad \left[\because A = x \times t_1 \ \therefore dA = dx \times t_1\right]$$

$$= \int_0^b \frac{F x h}{2 I_{xx}} \times (dx \times t_1)$$

$$= \frac{F h t_1}{2 I_{xx}} \int_0^b x \, dx$$

$$= \frac{F h t_1}{2 I_{xx}} \left[\frac{x^2}{2}\right]_0^b$$

$$= \frac{F h t_1 b^2}{4 I_{xx}}$$

$$\therefore \boxed{F^* = \frac{F h t_1 b^2}{4 I_{xx}}}$$

Step 3 : Location of shear centre.

Considering moments about point 'o', we have F as applied shear Force at a distance of 'b' from the centre of web.

$$F \times e = F^* \times h$$

$$\boxed{e = \frac{F^* \times h}{F}}$$

$$e = \frac{\cancel{F} h t_1 b^2}{4 I_{x-x}} \times \frac{h}{F}$$

$$\boxed{e = \frac{t_1 b^2 h^2}{4 I_{xx}}}$$

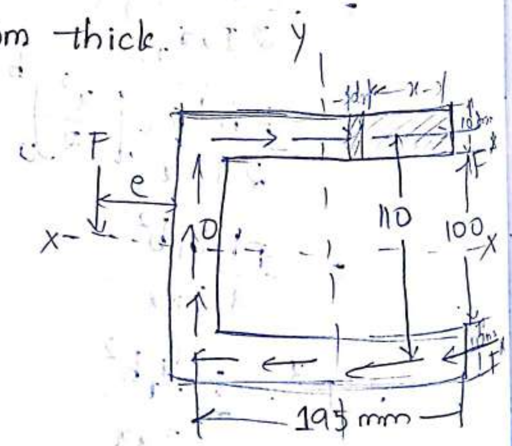
$$I_{xx} = 2\left[\frac{b \times t_1^3}{12} + b \times t_1 \times \left(\frac{h}{2}\right)^2\right] + \frac{t_2 h^3}{12}$$

Q) Determine the position of shear centre for a channel section of 120 mm × 120 mm Outside and 10 mm thick.

Given

Thickness = 10 mm

dimension = 120 × 120 mm



$t_1 = 10$ mm

$t_2 = 10$ mm

$b = 120$ mm

$h = 1100$ mm

wkt, $\tau = \frac{F A \bar{y}}{I_{xx} b_f}$

Area of shaded portion $= x \times t_1 = x \times 10 = 10x$

$$\tau = \frac{F \times (x \times 10) \times \frac{110}{2}}{I_{xx} \times 10}$$

$$\tau = \frac{F x 55}{I_{xx}}$$

2) Shear force produced in flanges

$$F^* = \tau \times dA$$

$$F^* = \int \tau \, dA$$

$$= \int_0^{120} \frac{55 F x}{I_{xx}} \times (dx \times 10)$$

$$= \frac{55 F \times 10}{I_{xx}} \int_0^{120} x \cdot dx$$

$$= \frac{55 F \times 10}{I_{xx}} \left[\frac{x^2}{2}\right]_0^{120}$$

$$F^* = \frac{550 F}{I_{xx}} \left(\frac{120^2}{2}\right)$$

3.025×10^6

0.472²

$$F^* = \frac{3.96 \times 10^6 F}{I_{xx}}$$

3) $F \times e = F^* \times h$

3) $e = \dfrac{F^* \times h}{F}$

$$e = \frac{3.96 \times 10^6 \cancel{F} \times 110}{I_{xx} \times \cancel{F}}$$

$$I_{xx} = 2\left[\frac{b \times t_1^3}{12} + b \times t_1 \times \left(\frac{h}{2}\right)^2\right] + \frac{t_2 h^3}{12}$$

$$= 2\left[\frac{120 \times 10^3}{12} + 120 \times 10 \times \left(\frac{110}{2}\right)^2\right] + \frac{10 \times 110^3}{12}$$

$$= 8.38 \times 10^6 \text{ mm}^4$$

$$e = \frac{3.96 \times 10^6 \times 110}{8.38 \times 10^6}$$

$$e = 51.98 \text{ mm}$$

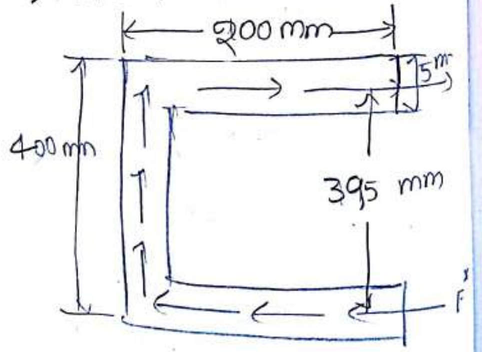
Q) Determine the position of shear center for a channel section of 400 mm × 200 mm outside and 5 mm thick.

Given

dimension of Section

= 400 × 200 mm

thickness = 5 mm



$t_1 = 5$ mm, $t_2 = 5$ mm

$b = 400$ mm, $h = 195$ mm

wkt, $\tau = \dfrac{F A \bar{y}}{I_{xx} b_f}$

$$\tau = \frac{F \times (x \times \cancel{5}) \times 197.5}{I_{xx} \times \cancel{5}}$$

$$\tau = \frac{197.5\, F x}{I_{xx}}$$

2) $F^* = \tau \times dA$

$$F^* = \int \tau\, dA$$

$$= \int_0^{200} \frac{197.5\, F x}{I_{xx}} (dx \times 5)$$

$$= \frac{197.5\, F \times 5}{I_{xx}} \int_0^{200} x \cdot dx$$

$$= \frac{197.5\, F \times 5}{I_{xx}} \left(\frac{x^2}{2}\right)$$

$$= \frac{197.5 \times 5 \times F}{I_{xx}} \left(\frac{200^2}{2}\right)$$

$$F^* = \frac{19.75 \times 10^6 F}{I_{xx}}$$

$$I_{xx} = 2\left\{\frac{200 \times 5^3}{12} + 200 \times 5 \times \left(\frac{395}{2}\right)^2\right\} + \frac{5 \times 395^3}{12}$$

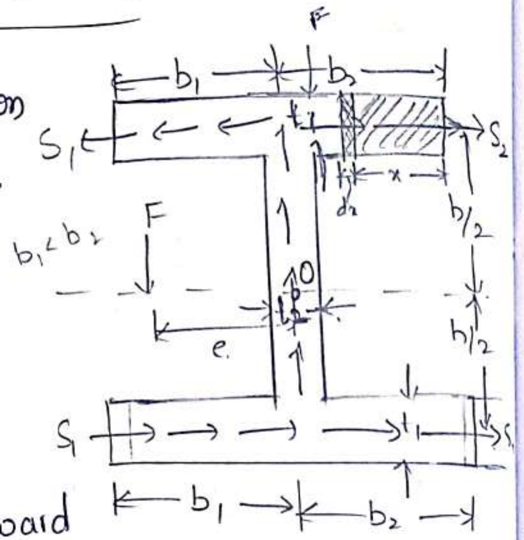
$$I_{xx} = 103.69 \times 10^6 \text{ mm}^4$$

3) $F \times e = F^* \times h$

$$e = \frac{19.75 \times 10^6 \times 395 \times \cancel{F}}{103.69 \times 10^6}$$

$$e = 75.23 \text{ mm}$$

Shear Centre for I-Section



The unequal I-section as shown in the fig. It is symmetrical about x-x axis, hence, the shear centre will lie on

let,

F = vertical downward applied shear force.

$b_1 + b_2$ = Total length of the Flange.

t_1 = thickness of the flange

t_2 = thickness of web.

S_1 = shear force in shorter length of the top flange

S_2 = shear force in longer length of top flange.

1) We know that $\tau = \frac{FA\bar{y}}{Ib}$

$$= \frac{F(x \times t_1) \times \frac{h}{2}}{I_{xx} \cdot t_1}$$

$$\tau = \frac{F x h}{2 I_{xx}}$$

where, F = applied shear force

$A\bar{y}$ = Moment of shaded area about axis of symmetry, i.e., about x-x axis.

$= (x \times t_1) \times \frac{h}{2}$

I = moment of Inertia about x-axis

b = width of the shaded section = t_1

Shear force in elementary area $dA = dx \times t_1$

S-2 $F^x = ds = \int \tau \times dA \Rightarrow \int \frac{Fxh}{2I_{xx}} \cdot dA$

∴ Total shear force in longer length of the top flange.

$$S_2 = \int_0^{b_2} \frac{Fxh}{2I_{xx}} (dx \times t_1)$$

$$= \frac{Fht_1}{2I_{xx}} \left[\frac{x^2}{2}\right]_0^{b_2}$$

$$S_1 = \frac{Fht_1 b_2^2}{4I_{xy}}$$

Similarly, Total shear force in shorter length of top flange

$$S_1 = \int_0^{b_1} \frac{Fxh}{2I_{xx}} (dx \times t_1) = \frac{Fht_1 b_1^2}{4I_{xx}}$$

$$S_1 = \frac{Fht_1 b_1^2}{4I_{xx}}$$

S-3 The couple due to S_2 will be clockwise whereas the couple due to S_1 will be anticlockwise. Also couple due to S_2 having larger value and hence net moment due to these couples will be clockwise. To balance this Net Moment the applied downward Shear Force F should act to the left of point 'O' on the x-axis to give anticlockwise moment

Take moments about 'O'

$$(S_2-S_1)\times h = e\times F$$

$$e = \frac{(S_2-S_1)\times h}{F}$$

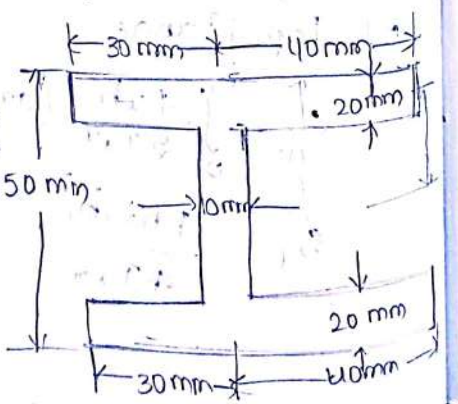
$$e = \frac{h^2 t_1 (b_2^2 - b_1^2)}{4I_{xx}}$$

I_{xy} formula is same as C-Section

Q). Determine the shear centre for unequal I section shown in fig.

$$I_{xx} = 2\left[\frac{70\times 20^3}{12} + 70\times 20\times\left(\frac{130}{2}\right)^2\right] + \frac{10\times 130^3}{12}$$

$$I_{xx} = 13.75\times 10^6 \ mm^4$$



$$\tau = \frac{FA\bar{y}}{Ib}$$

$$= \frac{F(x\times 20)\times 65}{13.75\times 10^6 \times 20}$$

$$\tau = 4.72\times 10^{-6} Fx$$

S-2

$$F^x = \int \frac{F x b}{2I_{xx}} dx$$

$$S_2 = \int_0^{40} F x\, x \qquad S_2 = 7.66\times 10^{-2}$$

$$S_1 = \int_0^{30} \frac{F x b}{2I_{xx}} dx$$

$$= \int_0^{30} \frac{F x 130}{2\times 13.75\times 10^{-6}} dx$$

$$= \frac{F\times 130}{2\times 13.75\times 10^{-6}} \int_0^{30} x\, dx$$

$$= 4.727\times 10^{-6} \left(\frac{30^2}{2}\right) \qquad S_1 = 0.0421 = 0.0\ 4.42\times 10^{-2}$$

$$e = 4.301 \ mm$$